

## Prenos varianc in kovarianc – Slepri poligon treh novih točk

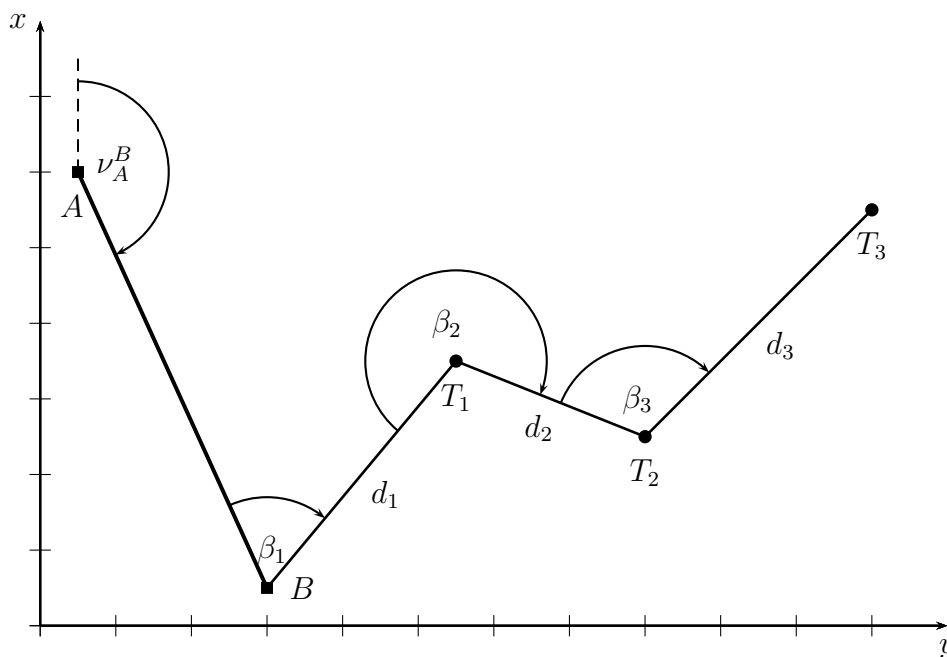
Določiti želimo koordinate treh novih točk ( $T_1$ ,  $T_2$  in  $T_3$ ) na osnovi dveh dani točk ( $A$  in  $B$ ) ter opazovanj slepega poligona ( $d_1$ ,  $\beta_1$ ,  $d_2$ ,  $\beta_2$ ,  $d_3$  in  $\beta_3$ ), kot to prikazuje slika 1. Dane koordinate točk  $A$  in  $B$  sta:

- $A(y_A, x_A) = (461\,300.0\text{ m}, 100\,600.0\text{ m})$  in
- $B(y_B, x_B) = (461\,400.0\text{ m}, 100\,550.0\text{ m})$ .

Opazovanja v slepem poligonu so enaka:

- $d_1 = 75.0\text{ m}$ ,  $\beta_1 = 100^\circ$ ,
- $d_2 = 50.0\text{ m}$ ,  $\beta_2 = 230^\circ$  in
- $d_3 = 100.0\text{ m}$ ,  $\beta_3 = 75^\circ$ .

Če je natančnost vseh dolžin enaka  $\sigma_d = 5.0\text{ cm}$  in je enaka tudi natančnost vseh kotov  $\sigma_\beta = 2'$ , izračunaj koordinate točk  $T_1(y_1, x_1)$ ,  $T_2(y_2, x_2)$  in  $T_3(y_3, x_3)$ . S prenosom varianc in kovarianc izračunaj tudi natančnosti vseh koordinat in vse korelacije med vsemi koordinatami.



Slika 1: Skica slepega poligona dveh danih in treh novih točk

1. Sestavimo vektor opazovanj  $\mathbf{x}$  in pripadajočo variančno-kovariančno matriko  $\Sigma_{xx}$ . Število opazovanj je 6, ki jih razvrstimo kot:

$$\mathbf{x} = [d_1 \quad \beta_1 \quad d_2 \quad \beta_2 \quad d_3 \quad \beta_3]^T \quad (1)$$

V vektorju  $\mathbf{x}$  iz enačbe 1 so dolžinske količine v metrih, kotne količine pa v radianih. Variančno-kovariančna matrika je velikosti  $6 \times 6$ , diagonalna, kjer so na diagonali elementi:

$$\Sigma_D = \begin{bmatrix} \sigma_d^2 & \sigma_\beta^2 & \sigma_d^2 & \sigma_\beta^2 & \sigma_d^2 & \sigma_\beta^2 \end{bmatrix} \quad (2)$$

Vrednost varianc iz enačbe 2 sta  $\sigma_d^2 = 0.0025 \text{ m}^2$  in  $\sigma_\beta^2 = 3.385 \times 10^{-7}$ .

2. Določimo vse naše neznanke  $y_j$  ( $j = 1, \dots, m$ ) in sestavimo vektor neznank  $\mathbf{y}$ . Določiti želimo koordinate treh točk novih točk ( $T_1(y_1, x_1)$ ,  $T_2(y_2, x_2)$  in  $T_3(y_3, x_3)$ ), torej je  $m = 6$  in velja:

$$\mathbf{y} = \begin{bmatrix} y_1 & x_1 & y_2 & x_2 & y_3 & x_3 \end{bmatrix}^T \quad (3)$$

3. Določimo funkcijske zveze med neznankami in opazovanji,  $y_j = f_j(x_1, x_2, x_3, \dots, x_n)$ , ( $j = 1, \dots, m$ ) in izračunamo vrednosti neznank  $\mathbf{y}$ .

Uporabimo enačbe za izračun točk v slepem poligonu. Velja:

$$\begin{aligned} y_1 &= y_B + d_1 \sin(\nu_B^{T_1}) = 461\,444.680 \text{ m} \\ x_1 &= x_B + d_1 \cos(\nu_B^{T_1}) = 100\,610.239 \text{ m} \\ y_2 &= y_B + d_1 \sin(\nu_B^{T_1}) + d_2 \sin(\nu_{T_1}^{T_2}) = 461\,494.590 \text{ m} \\ x_2 &= x_B + d_1 \cos(\nu_B^{T_1}) + d_2 \cos(\nu_{T_1}^{T_2}) = 100\,613.234 \text{ m} \\ y_3 &= y_B + d_1 \sin(\nu_B^{T_1}) + d_2 \sin(\nu_{T_1}^{T_2}) + d_3 \sin(\nu_{T_2}^{T_3}) = 461\,462.968 \text{ m} \\ x_3 &= x_B + d_1 \cos(\nu_B^{T_1}) + d_2 \cos(\nu_{T_1}^{T_2}) + d_3 \cos(\nu_{T_2}^{T_3}) = 100\,708.103 \text{ m} \end{aligned} \quad (4)$$

V enačbi 4 smerne kote med točkami ( $\nu_B^{T_1}$ ,  $\nu_{T_1}^{T_2}$  in  $\nu_{T_2}^{T_3}$ ) izračunamo kot:

$$\begin{aligned} \nu_B^{T_1} &= \nu_A^B + \beta_1 - 180^\circ = 36^\circ 33' 54'' \\ \nu_{T_1}^{T_2} &= \nu_A^B + \beta_1 + \beta_2 - 2 \cdot 180^\circ = 86^\circ 33' 54'' \\ \nu_{T_2}^{T_3} &= \nu_A^B + \beta_1 + \beta_2 + \beta_3 - 3 \cdot 180^\circ = 341^\circ 33' 54'' \end{aligned} \quad (5)$$

4. Izračunamo vseh  $m \times n$  parcialnih odvodov  $\frac{\partial f_j}{\partial x_i}$  in sestavimo Jakobijevo matriko  $\mathbf{J}$  velikosti  $m \times n$ .

Izračunati moramo vse parcialne odvode, vseh 6 neznank moramo odvajati o vseh 6-ih opazovanjih. Imamo torej 36 parcialnih odvodov, ki pa imajo obliko:

- parcialni odvodi  $y_1$ :

$$\begin{aligned} \frac{\partial y_1}{\partial d_1} &= \sin(\nu_B^{T_1}) & \frac{\partial y_1}{\partial \beta_1} &= d_1 \cos(\nu_B^{T_1}) \\ \frac{\partial y_1}{\partial d_2} &= 0 & \frac{\partial y_1}{\partial \beta_2} &= 0 \\ \frac{\partial y_1}{\partial d_3} &= 0 & \frac{\partial y_1}{\partial \beta_3} &= 0 \end{aligned} \quad (6)$$

- parcialni odvodi  $x_1$ :

$$\begin{aligned}\frac{\partial x_1}{\partial d_1} &= \cos(\nu_B^{T_1}) & \frac{\partial x_1}{\partial \beta_1} &= -d_1 \sin(\nu_B^{T_1}) \\ \frac{\partial x_1}{\partial d_2} &= 0 & \frac{\partial x_1}{\partial \beta_2} &= 0 \\ \frac{\partial x_1}{\partial d_3} &= 0 & \frac{\partial x_1}{\partial \beta_3} &= 0\end{aligned}\tag{7}$$

- parcialni odvodi  $y_2$ :

$$\begin{aligned}\frac{\partial y_2}{\partial d_1} &= \sin(\nu_B^{T_1}) & \frac{\partial y_2}{\partial \beta_1} &= d_1 \cos(\nu_B^{T_1}) + d_2 \cos(\nu_{T_1}^{T_2}) \\ \frac{\partial y_2}{\partial d_2} &= \sin(\nu_{T_1}^{T_2}) & \frac{\partial y_2}{\partial \beta_2} &= d_2 \cos(\nu_{T_1}^{T_2}) \\ \frac{\partial y_2}{\partial d_3} &= 0 & \frac{\partial y_2}{\partial \beta_3} &= 0\end{aligned}\tag{8}$$

- parcialni odvodi  $x_2$ :

$$\begin{aligned}\frac{\partial x_2}{\partial d_1} &= \cos(\nu_B^{T_1}) & \frac{\partial x_2}{\partial \beta_1} &= -d_1 \sin(\nu_B^{T_1}) - d_2 \sin(\nu_{T_1}^{T_2}) \\ \frac{\partial x_2}{\partial d_2} &= \cos(\nu_{T_1}^{T_2}) & \frac{\partial x_2}{\partial \beta_2} &= -d_2 \sin(\nu_{T_1}^{T_2}) \\ \frac{\partial x_2}{\partial d_3} &= 0 & \frac{\partial x_2}{\partial \beta_3} &= 0\end{aligned}\tag{9}$$

- parcialni odvodi  $y_3$ :

$$\begin{aligned}\frac{\partial y_3}{\partial d_1} &= \sin(\nu_B^{T_1}) & \frac{\partial y_3}{\partial \beta_1} &= d_1 \cos(\nu_B^{T_1}) + d_2 \cos(\nu_{T_1}^{T_2}) + d_3 \cos(\nu_{T_2}^{T_3}) \\ \frac{\partial y_3}{\partial d_2} &= \sin(\nu_{T_1}^{T_2}) & \frac{\partial y_3}{\partial \beta_2} &= d_2 \cos(\nu_{T_1}^{T_2}) + d_3 \cos(\nu_{T_2}^{T_3}) \\ \frac{\partial y_3}{\partial d_3} &= \sin(\nu_{T_2}^{T_3}) & \frac{\partial y_3}{\partial \beta_3} &= d_3 \cos(\nu_{T_2}^{T_3})\end{aligned}\tag{10}$$

- parcialni odvodi  $x_3$ :

$$\begin{aligned}\frac{\partial x_3}{\partial d_1} &= \cos(\nu_B^{T_1}) & \frac{\partial x_3}{\partial \beta_1} &= -d_1 \sin(\nu_B^{T_1}) - d_2 \sin(\nu_{T_1}^{T_2}) - d_3 \sin(\nu_{T_2}^{T_3}) \\ \frac{\partial x_3}{\partial d_2} &= \cos(\nu_{T_1}^{T_2}) & \frac{\partial x_3}{\partial \beta_2} &= -d_2 \sin(\nu_{T_1}^{T_2}) - d_3 \sin(\nu_{T_2}^{T_3}) \\ \frac{\partial x_3}{\partial d_3} &= \cos(\nu_{T_2}^{T_3}) & \frac{\partial x_3}{\partial \beta_3} &= -d_3 \sin(\nu_{T_2}^{T_3})\end{aligned}\tag{11}$$

Zgornje parcialne odvode vstavimo v jakobijevo matriko  $\mathbf{J}$  in dobimo:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial y_1}{\partial d_1} & \frac{\partial y_1}{\partial \beta_1} & \frac{\partial y_1}{\partial d_2} & \frac{\partial y_1}{\partial \beta_2} & \frac{\partial y_1}{\partial d_1} & \frac{\partial y_1}{\partial \beta_3} \\ \frac{\partial x_1}{\partial d_1} & \frac{\partial x_1}{\partial \beta_1} & \frac{\partial x_1}{\partial d_2} & \frac{\partial x_1}{\partial \beta_2} & \frac{\partial x_1}{\partial d_1} & \frac{\partial x_1}{\partial \beta_3} \\ \frac{\partial y_2}{\partial d_1} & \frac{\partial y_2}{\partial \beta_1} & \frac{\partial y_2}{\partial d_2} & \frac{\partial y_2}{\partial \beta_2} & \frac{\partial y_2}{\partial d_1} & \frac{\partial y_2}{\partial \beta_3} \\ \frac{\partial x_2}{\partial d_1} & \frac{\partial x_2}{\partial \beta_1} & \frac{\partial x_2}{\partial d_2} & \frac{\partial x_2}{\partial \beta_2} & \frac{\partial x_2}{\partial d_1} & \frac{\partial x_2}{\partial \beta_3} \\ \frac{\partial y_3}{\partial d_1} & \frac{\partial y_3}{\partial \beta_1} & \frac{\partial y_3}{\partial d_2} & \frac{\partial y_3}{\partial \beta_2} & \frac{\partial y_3}{\partial d_1} & \frac{\partial y_3}{\partial \beta_3} \\ \frac{\partial x_3}{\partial d_1} & \frac{\partial x_3}{\partial \beta_1} & \frac{\partial x_3}{\partial d_2} & \frac{\partial x_3}{\partial \beta_2} & \frac{\partial x_3}{\partial d_1} & \frac{\partial x_3}{\partial \beta_3} \end{bmatrix} = \begin{bmatrix} 0.596 & 60.239 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.803 & -44.680 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.596 & 63.234 & 0.998 & 2.996 & 0.000 & 0.000 \\ 0.803 & -94.590 & 0.060 & -49.910 & 0.000 & 0.000 \\ 0.596 & 158.103 & 0.998 & 97.864 & -0.316 & 94.868 \\ 0.803 & -62.968 & 0.060 & -18.287 & 0.949 & 31.623 \end{bmatrix} \quad (12)$$

5. Izračunamo kovariančno matriko neznank  $\Sigma_{yy} = \mathbf{J}\Sigma_{xx}\mathbf{J}^T$ .

Izračun variančno-kovariančne matrike  $\Sigma_{yy}$  sledi po sestavi variančno-kovariančne matrike  $\Sigma_{xx}$  in jakobijeve matrike  $\mathbf{J}$ , izpis matrike pa bomo tu, zaradi velikosti matrike, izpustili. Vsi rezultati so podani v nadaljevanju.

6. Iz variančno-kovariančne matrike neznank  $\Sigma_{yy}$  izračunamo natančnosti neznank  $\sigma_j$  ( $j = 1, \dots, m$ ) in korelacije med neznankami  $\rho_{i,j}$  ( $i, j = 1, \dots, m \wedge i \neq j$ ).

Izračunana natančnost koordinat za vsako točko in pripadajoča korelacija so:

$$\begin{aligned} \sigma_{y_1} &= 0.046 \text{ m} & \sigma_{x_1} &= 0.048 \text{ m} & \rho_{y_1x_1} &= 0.130 \\ \sigma_{y_2} &= 0.069 \text{ m} & \sigma_{x_2} &= 0.074 \text{ m} & \rho_{y_2x_2} &= -0.143 \\ \sigma_{y_3} &= 0.136 \text{ m} & \sigma_{x_3} &= 0.075 \text{ m} & \rho_{y_3x_3} &= -0.232 \end{aligned} \quad (13)$$

Vse ostale korelacije lahko zapišemo v pregledni obliki:

|       | $y_1$ | $x_1$ | $y_2$ | $x_2$ | $y_3$ | $x_3$ |
|-------|-------|-------|-------|-------|-------|-------|
| $y_1$ | 1.00  |       |       |       |       |       |
| $x_1$ | 0.13  | 1.00  |       |       |       |       |
| $y_2$ | 0.69  | 0.07  | 1.00  |       |       |       |
| $x_2$ | -0.21 | 0.86  | -0.14 | 1.00  |       |       |
| $y_3$ | 0.66  | -0.18 | 0.74  | -0.53 | 1.00  |       |
| $x_3$ | -0.03 | 0.71  | -0.00 | 0.71  | -0.23 | 1.00  |

Korelacije v zgornji preglednici so podane za vse možne kombinacije neznank. V sivem so korelacije neznanke s samimi seboj, zato vedno vrednost 1. Modre so korelacije med koordinatama iste točke.