Introduction to Gravimetry

Conducting and Processing Relative Gravity Surveys

- A Brief Tutorial -

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1. Calibration of the Gravity Meter

1.1 Calibration Function and Calibration Table

The measurements are usually read in *counter units* (C. U.) and have to be converted to gravity units (usually $\mu m/s^2$). For LACOSTE-ROMBERG gravity meters one counter unit roughly corresponds to one mGal[1], but not precisely. The exact relation between counter unit measurements and gravity values is given by a *calibration function* which is normally expressed time-independently:

$$g = f(z) \tag{1.1}$$

The calibration function can be separated into a polynomial part representing the long wave components and a periodic part modeling the error of the dial via a FOURIER series and is to be applied for surveys of higher accuracy:

$$f_{polynomial}(z) = \sum_{n=0}^{m} p_n \cdot z^n \tag{1.2}$$

$$f_{periodic}(z) = \sum_{i=1}^{l} c_i \cdot \cos(\omega_i \cdot z) + s_i \cdot \sin(\omega_i \cdot z) = \sum_{i=1}^{l} a_i \cdot \cos(\omega_i \cdot z - \varphi_i) \tag{1.3}$$

p_n are polynomial coefficients, c_i and s_i are amplitudes of cosine and sine terms, respectively, and a_i as well as φ_i denote the same function using the amplitude/phase angle-representation. In this way we yield a calibration function of the form $f(z) = f_{polynomial}(z) + f_{periodic}(z)$.

However, the manufacturer calibrates the device before it is purchased and thereby provides a calibration table that is used to convert counter units into gravity units. For LCR-D gravity meters the calibration table can be replaced by a single scale factor. The calibration table for the LCR-G688 used at the Institute of Geodesy is given below. The interpolation procedure is as follows [📖 6]:

2. Read counter and dial (for example 512.36 C. U.).
3. Choose the value from the calibration table which is the nearest lower neighbor to your value and take the actual value (500 C. U. → 493.85 mGal).
4. Compute the difference between the original counter reading and the counter reading for the start interval (512.36-500.00 → 12.36), multiply it by the interval factor given in the table (12.36 x 0.98701 → 12.20) and add this value to the actual value chosen before: 493.85 + 12.20 → 506.05 mGal = 5060.5 $\mu m/s^2$.

Counter Reading	Value [mGal]	Factor for Interval
0.0	0.00	0.98723
100.0	98.72	0.98714
200.0	197.44	0.98707
300.0	296.14	0.98704
400.0	394.85	0.98701
500.0	493.85	0.98701
600.0	592.25	0.98701
700.0	690.95	0.98701
800.0	789.65	0.98702

Table 1: *Some lines of the calibration table for the LaCoste-Romberg Relative Gravity Meter No. G688*

[1]) 1 mGal corresponds to 0.1 $\mu m/s^2$ and is commonly used in gravimetry. Nevertheless, in this tutorial, the SI-based unit $\mu m/s^2$ is used.

There might be, of course, variations over time that require a new calibration of the device. Moreover, for high precision measurements the manufacturer's calibration information has to be improved or, at least, confirmed. In this way, we may consider the calibration table as an approximated calibration function $f_0(z)$ that yields approximately calibrated readings $\tilde{z} = f_0(z)$. The main task of a new calibration is the determination of the *deviations* from this approximate function. So, the calibration function can be expressed - analogously to the first paragraph - as $f(z) = f_0(z) + \Delta f(z)$ where the last term $\Delta f(z) = \Delta f_{polynomial}(z) + \Delta f_{periodic}(z)$ models the deviations from the approximate function and is to be determined by the user. If highest accuracy is not required, the periodic errors can be neglected (depends on gravity meter) and it may be sufficient to determine the linear error, i. e. higher degree coefficients of the polynomial are neglected as well.

1.2 Determination of the Calibration Function

Several laboratory methods have been tested to determine the calibration function. However, the most common approach is the use of *calibration lines*. There are lines covering large gravity differences or smaller ones and the user has to decide which one meets his requirements. It should be noted that any higher order gravity network may be used to calibrate relative gravimeters if its accuracy and extension are suitable. The map on the right shows the calibration line from Hannover to the Harz Forest.

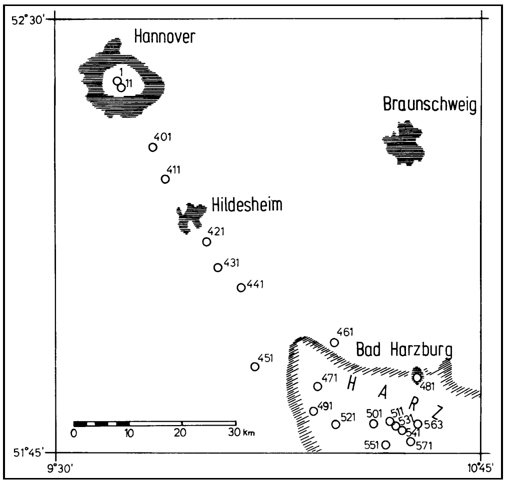

Figure 1: *Calibration Line Hannover - Harz, Germany* [📖 1

Obviously, the accuracy of the linear calibration coefficient[2] p_1 depends on the gravity range - or, equivalently, counter reading range - covered:

$$p_1 = \frac{\Delta g}{\Delta z} \quad \Rightarrow \quad \sigma_{p_1} = \frac{1}{\Delta z} \cdot \sqrt{\sigma_{\Delta g}^2 + \frac{\Delta g^2}{\Delta z^2} \cdot \sigma_{\Delta z}^2} \qquad 1.4$$

The line Harzburg - Torfhaus has a gravity difference of 860 µm/s². This yields an accuracy of 0.00016 = 0.16‰ if a precision of 0.1 µm/s² can be assumed for the gravity difference as well as for the counter reading difference. Taking the difference between München and Hamburg of 6,400 µm/s² with a standard deviation of 0.1 µm/s² and a standard deviation of 0.15 µm/s² for the measurement, the resulting precision of the scale turns out to be 0.03‰.

In these two examples, the gravity change is mainly caused by a change in latitude. Alternatively, calibration systems may also be established exploiting the decrease in gravity with increasing height. High-rise buildings can be used (skyscrapers), although, in this case, high precision measurements may suffer from wind shocks.

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[2]) The so-called *scale factor* is the most important coefficient of the calibration function.

2. Planning and Conducting the Survey

2.1 Networks and Selection of Control Points

Gravity control points serve as a frame for subsequent detailed surveys. The following types of networks can be distinguished [🕮 3]:

- *global* gravity networks with station separations of a few 100 up to 1,000 km. They are the basic elements of gravity reference systems and are established in international cooperation;
- *regional* gravity networks with station separations of a few km to 100 km. They are established as national networks mostly in the form of a fundamental gravity network with related densification networks;
- *local* gravity networks with station separations of a few 0.1 to 10 km. These are mostly established for geophysical or geodynamical purposes.

Some general aspects for the distribution and local selection of gravity control points may be stated as follows:

- control point distribution as uniform as possible over the survey area, exceptions may be local geodynamic networks;
- local geological, seismic, and hydrological stability;
- stable location for instrument setup (ground points in buildings, pillar, rock, concrete floor), monumentation of the measurement point; it is expedient to utilize already existing markers of horizontal or vertical control points;
- establishment of 2 to 3 gravimetric eccenters (±0.01 to 0.1 $\mu m/s^2$) for verification of station integrity in fundamental gravity networks (eccenter separations a few 100 m up to a few km; gravity differences smaller than 100 $\mu m/s^2$);
- horizontal and vertical position determination relative to national control networks.

2.2 Station Description

All gravity points are necessarily to be well documented, especially for retrieval purposes. TORGE [🕮 3] gives the following advice as far as station documentation is concerned:

Station descriptions for gravity control points consist of a *verbal* and a *pictorial* part. The verbal part should contain the following information:

- station number, station name (name of site);
- description of point location and monumentation;
- geographical latitude and longitude (±0.1") and plane coordinates (±1 m) in the national reference system;
- station elevations (± mm) in the national reference system;
- identity with points of other gravity networks.

It is useful to state an approximate gravity value (± $\mu m/s^2$). For gravity base networks the address of the supervising organization should be included. Finally, station-specific information (earth tide parameters, local gravity gradient, geological and hydrological information) should be given if available.

The pictorial part should comprise:

- photographs of the measurement point and its dose proximity;
- overview and in-survey sketches with data of local control measures;
- extract of large scale map, 1:2,500 or similar (cadastral map, city map);
- extract of topographic map, 1:50,000 or similar.

Photographs, survey sketches, and large scale map extract should focus particularly on control of changes of the measurement station.

The requirements for station descriptions are less stringent for gravity points in densification networks and for regional and local surveys. A sketch of the in-survey and a mark in a topographic map should always be completed.

Figure 2 shows two examples for extracts of station documentation as well as sketches.

2.3 Preparation of the Survey

Prior to the survey of a network, the gravimeters have to undergo instrumental investigations and calibrations. These include:

- laboratory tests with respect to dependencies on atmospheric pressure, temperature, magnetic field, shocks, and others;
- investigations of the drift behavior of spring gravimeters;
- calibration of spring gravimeters in a calibration system with a range that exceeds the gravity range of the survey area;
- comparisons of different instruments by parallel surveys of the same points or the same gravity differences, respectively.

These investigations lead to

- a-priori estimates of achievable measurement precision;
- indications of systematic error sources which have to be taken into account by modeling or by instrumental measures;

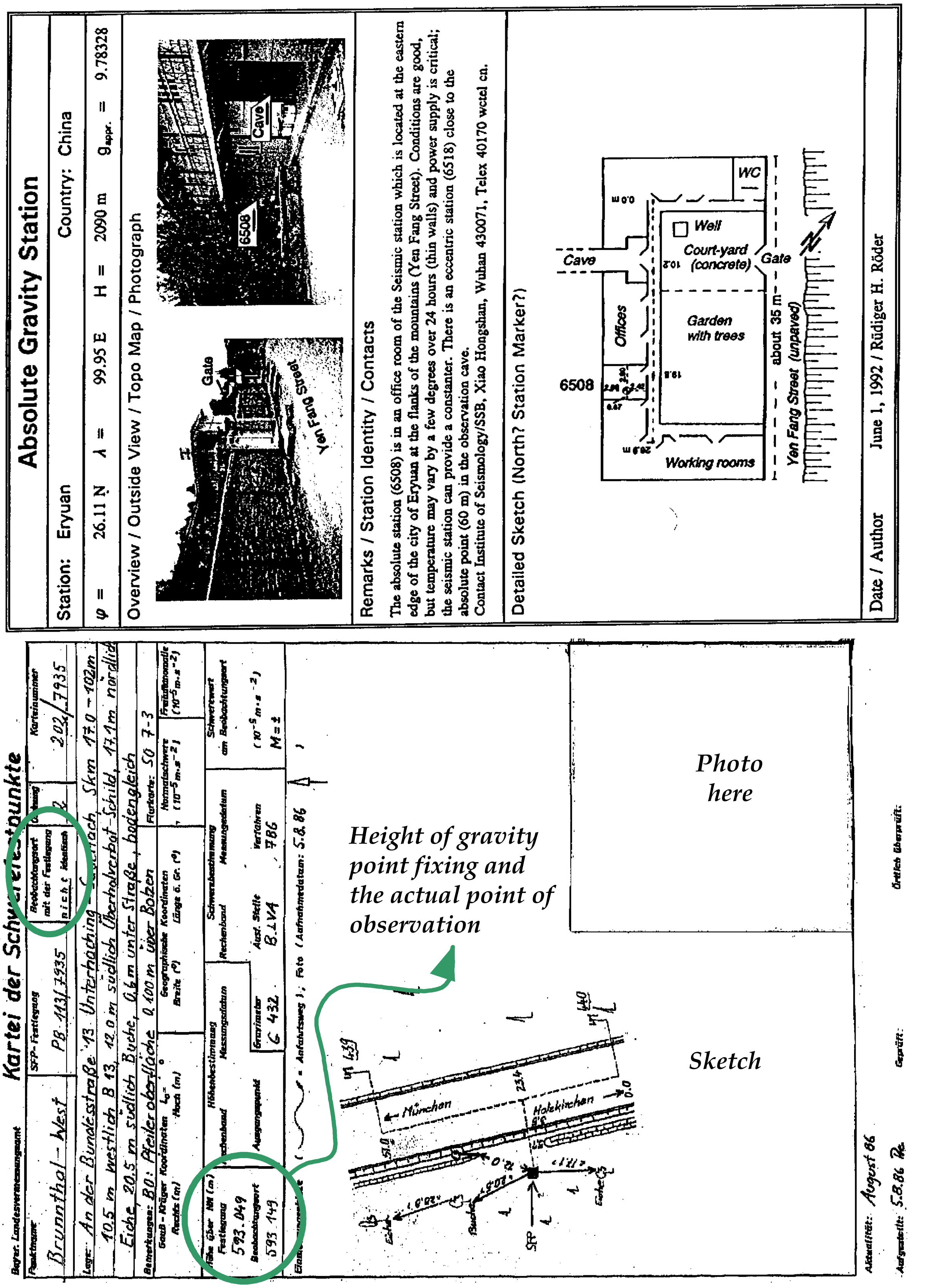

Figure 2: *Two examples for gravity station descriptions and sketches. If the fixing of the gravity point is not identical with the observation point like in the second example, this fact is to be clearly stressed (see marked entries!).*

2.4 Accomplishment of the Survey

2.4.1 Survey Procedure and Drift Control

Sub-networks are surveyed with relative gravimeters whereby network stations are often located on loops between stations of the *fundamental* network, and surveyed in lines using profile or step methods. A drift control should be planned in intervals of a few hours.

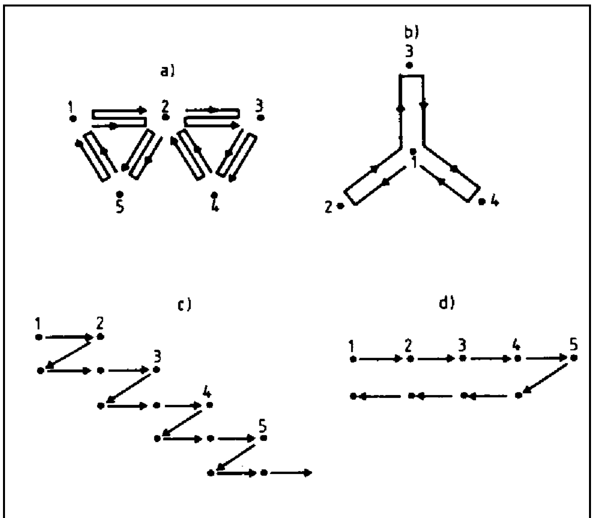

Figure 3: *Typical drift determination methods are the* ***difference method*** *(a) with three runs per line, the* ***star method*** *(b), the* ***step method*** *(c, station will be occupied three times) and the* ***profile method*** *(d, double occupation)* [📖 3]. *The user has to decide which one to choose with special respect to accuracy and efficiency. The difference method, for example, offers the advantage of high redundancy and, thus, one may expect a good accuracy, but it is economically very inefficient. The star method, in contrast, is more efficient. However, it might become inconvenient to return to the starting point each time. The profile method offers drift determination in forward-backward manner and might be good overall compromise.*

2.4.2 Azimuth Dependency

Spring gravimeters like LACOSTE-ROMBERG relative gravity meters (G and D) show an azimuth-dependency. This effect has to be eliminated by aligning the gravimeter to northward direction (use compass) or, if north direction cannot be determined properly, by always aligning the gravity meter with the same direction. Figure 4 shows the azimuth-dependency for a LCR-G gravity meter. The amplitude of the magnetic influence is about 0.04 μm/s².

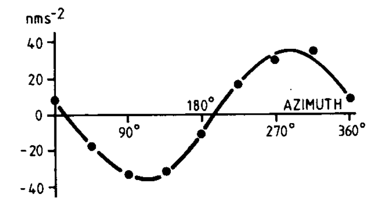

Figure 4: *Azimuth-dependent magnetic effect on gravimeter reading of LCR-G298.* [📖 3]

2.4.3 Transport

The covariance function of the counter readings might reveal a correlation due to several effects. One effect causing such correlation can be an incomplete compensation of the gravimeter drift, but, in practice, the drift is accounted for using polynomials or other models and the influence of the remainders does not cause any strong correlation (see Figure 5). However, the *conditions of transportation* can have an impact on the variance and covariance: The standard deviation of the readings increases with time, see [📖 5],

$$\sigma_{\Delta l} = \sqrt{2 \cdot C_{\Delta t=0} - 2 \cdot C_{\Delta t}} \qquad 2.1$$

where C_i are the time-dependent values of the covariance function. The following figures show the covariance functions with respect to the gravimeter drift and the increase in standard deviation for different kinds of transportation. Transportation by foot apparently yields most accurate results.

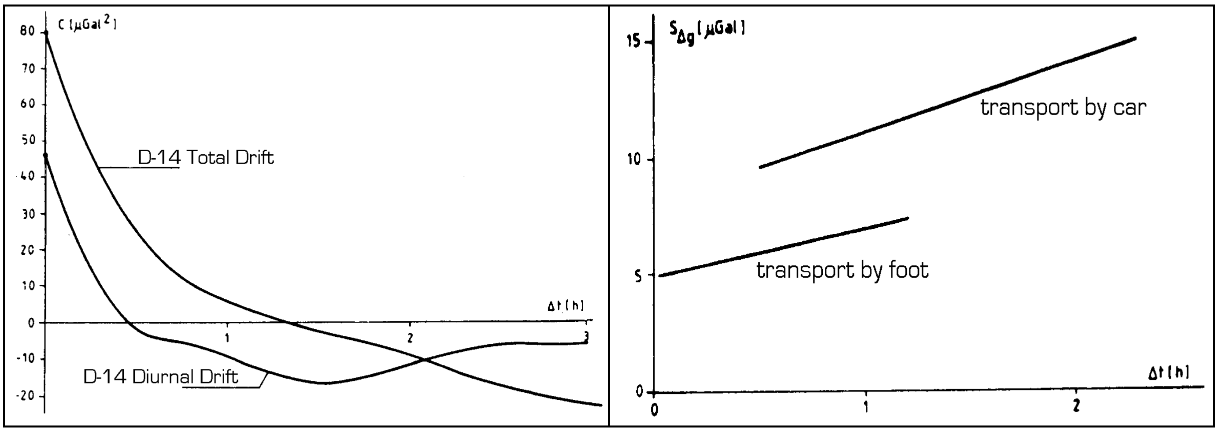

Figure 5 and Figure 6: *Covariance function of the gravity meter readings for the LCR-D14 (left) and the standard deviation of LCR gravimeters for observed gravity differences (right).* [📖 5]

2.4.4 Counter Reading

The operation of the LCR gravimeter can be summarized as follows: Place the gravimeter on the observation point, turn on the light for the levels and the optical system and level the gravity meter. Now, turn the arrestment knob in counterclockwise direction in order to release the spring. The beam can move now and you may monitor its motion in the telescope. The reading operation continues according to [📖 6]:

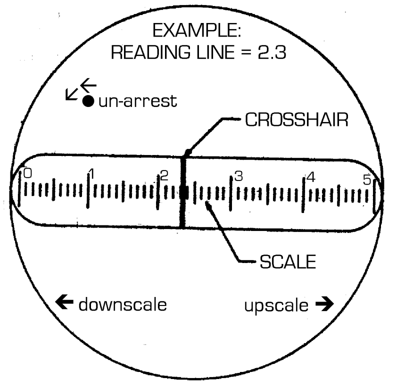

Figure 7: *View as seen in eyepiece of LCR gravity meters.*

- The downscale side of the crosshair is brought to the reading line[3] by turning the measuring screw. In order to move the crosshair upscale, you have to turn the measuring screw clockwise. Note that the reading accuracy will be enhanced if you approach the reading line by turning the screw in the same direction for all gravity readings. Avoid any backslash.
- Obtain the meter reading from the counter and the dial. The last digit on the counter and the number on the dial should correspond. *Example:* Counter units read 26543 and dial setting 36 yields 2654.36 C. U.

Do not forget to turn the arrestment knob clockwise again after your measurements are finished on the station!

2.5 Achievable Accuracy

The achievable measurement accuracy strongly depends on external conditions. Unfavorable measurement conditions (extremely high or deep temperatures, large temperature variations, wind, strong microseismics, bad transportation conditions, and other) can reduce the accuracy to such an extent that additional measurements

[3]) The reading line is given on the gravity meter.

may be required. Sometimes quality analysis may be useful to back-up respective decisions. System-specific systematic effects partly cancel if the same measurement schedule, i.e. sequence and timing, is maintained on each station. The following tables give an overview of the error budget [📖 3]:

Tab. 2 : Error budget for LCR gravimeters (gravity difference observed once, average measurement conditions, $\Delta g < 1000\,\mu ms^{-2}$), after Torge (1984)
*) Normal procedure: calibration table of manufacturer + linear calibration factor from calibration line,
**) Sum of squared error components.

Error Source	Error in Normal Procedure (μms^{-2})	Precision Gravimetry Measure	Precision Gravimetry Error (μms^{-2})
instrument			
reading	±0.03	external voltmeter	±0.01
leveling	±0.05	improved levels, fixed waiting time	±0.02
elastic aftereffects	±0.05	after unclamping	±0.01
voltage source	±0.05	voltage stabilizer	±0.01
calibration *)			
long-wave components	±0.15/ $1000\mu ms^{-2}$	calibration system with absolute values and subdivisions	±0.05/ $1000\mu ms^{-2}$
periodic components		special calibration line and/or feed-back calibration	
– Model G	±0.15		±0.01
– Model D	±0.02		±0.01
external effects			
temperature	±0.1	small temp. changes or additional thermostat	±0.02
air pressure	±0.01	control, poss. correct.	±0.01
magnetic field	±0.03	control, poss. correct.	±0.01
shocks	±0.1	careful transportation or additional transport container	±0.05
temporal gravity changes			
gravimetric tides	±0.1	tidal models for elastic earth and oceans or observed tidal param.	±0.01
air pressure fluctuations	±0.05	correction to standard air pressure	±0.005
fluctuations of ground water level and soil humidity	±0.05	(correction to standard ground water level not yet possible)	±0.05
standard deviation of gravity difference **)			
random error comp.	±0.17		±0.06
random + systematic error component			
– Model G	±0.27		±0.08
– Model D	±0.22		±0.08
total error incl. temporal gravity changes			
– Model G	±0.30		±0.10
– Model D	±0.25		±0.10

Tab. 3 : Standard deviations of a gravity difference Δg observed *once* with LCR gravimeters (Models G and D)

Project	Average Δg (μms^{-2})	Standard Deviation (μms^{-2})	Remarks	References
Vertical Calibration Line Hannover	<10	G,D: ±0.05	station separations few m	Kanngieser et al. 1983a
Fennoscand. Land Uplift Line	<6	G:±0.04…0.18 G,D: ±0.11	station separations up to 200 km	Kiviniemi 1974 Groten and Becker 1983
Calibration line Hannover–Harz	<100	G:±0.1…0.15 D: ±0.12 D:±0.03…0.14	station separations 5…15 km	Kanngieser et al.1983a Becker 1984
Control Network North Iceland	10…1000	G,D:±0.1…0.2	difficult conditions	Kanngieser 1982a
Fundamental Gravity Network Fed. Rep. of Germany	100…2000	G:±0.07…0.21	station separations 100…200km	Sigl et al. 1981
Ties Continent – Iceland	10000	G:±0.2…0.8	airplane transp., diff. centerings	Torge and Kanngieser 1983
Regional Network Europe–Africa	1000…20000	G:±0.3…0.8	insufficient drift control	Torge 1971

3. Pre-Processing of Data

Pre-processing of the data means computing the mean values per station, applying corrections and reductions and cleaning the data set. Some steps are explained here.

Instrumental Correction

First, the calibration correction is applied to the raw measurements as explained in chapter 1.

Height Reduction

The height reduction can easily cause errors than can hardly be detected if not applied correctly. The figure on the right shows the heights and height differences needed to compute correct height reductions. Note that the height of the fixing of the point H_{Point} is not necessarily identical with the height where the observation is carried out (lineup), H_{Bottom}. The height from the gravity meter bottom to the upper edge, DH_{total}, is measured by the user (usually somewhere near 21 cm), but the vertical position of the internal reference point, indicated by DH_{sys} (about 15,5 cm for the LCR-G688), is also to be taken into account. This yields

$$DH_{red} = H_{Bottom} + DH_{total} - DH_{sys} - H_{Point} \qquad 3.1$$

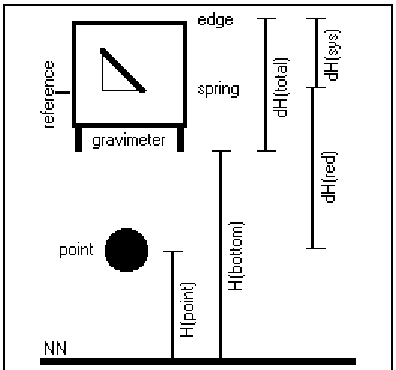

Figure 8: *Graphical illustration of how to compute the height needed for the height reduction.*

This height difference is converted into a gravity change by applying the free air gradient. If no local values are available, the standard value of -3.086 µm/s² per m in vertical direction can be used. Consequently, the gravity correction is:

$$\delta g_h = 3.086 \left[\mu m s^{-2} / m \right] \cdot \Delta H_{red} \qquad 3.2$$

Pressure Reduction

Variations in atmospheric pressure can cause gravity changes in the range of -3 to -4 nm/s² per hPa. Therefore, this reduction is to be applied if the deviation from the standard pressure p_n that can be computed to

$$p_n = 1013.25 \,[hPa] \cdot \left(1 - \frac{0.0065 \left[1/m \right] \cdot H}{288.15} \right)^{5.2559} \qquad 3.3$$

becomes larger and if there are strong pressure variations (e.g. due to huge height differences). The value of reduction can be estimated by regression analysis at the site. Usually, no local regression coefficients are available. In this case, one may use a standard factor of -3 nm/s² per hPa. The reduction becomes:

$$\delta g_p = 3 \left[nm s^{-2} / hPa \right] \cdot (p - p_n) \qquad 3.4$$

Tidal Reduction

Tidal reductions can be determined using several models. The expansion of the tidal potential for the rigid earth by CARTWRIGHT-TAYLER-EDDEN was recommended by the IAG in 1971. This model is usually sufficient for field surveys. For highest precision, newer developments can be deployed. However, in this case it becomes more important to use individual (local or regional) amplitude factors and phase angle shifts from observations at earth tide stations whereas in the normal case, a gravimetric factor of 1.16 is a good choice.

Several potential developments have been conducted so far, for example:

Year	*No. of waves*	*Developed by*
1921	378	DOODSON
1973	505	CARTWRIGHT-TAYLER-EDDEN
1985	665	BUELLESFELD
1987	1,200	TAMURA
1989	2,300	XI
1995	12,935	HARTMANN AND WENZEL

Table 2: *Overview of tidal potential developments.*

Ocean Loading Effects

Ocean loading is usually neglected, but for higher precision it may be necessary to take it into account. In practice, one possibility to do so is the use of processed amplitudes and phase angles for the main tidal waves. The following excerpt shows the results of such an analysis for Matera:

```
$$ Ocean loading gravity
$$ COLUMN ORDER:  M2  S2  N2  K2  K1  O1  P1  Q1  MF  MM SSA
$$ ROW ORDER:
$$ AMPLITUDES (0.1 mGal = 1 um/s^2)
$$ PHASES (deg)
$$
  MATERA   12734
$$ CSR3_PP ID: Jul   2, 1997 13:11
$$ Computed by OLMPP by H G Scherneck, Onsala Space Observatory, 1997
$$ MATERA       12734,  12734     GRAV lon/lat:    16.7047   40.6488
  .01127 .00433 .00215 .00114 .00299 .00147 .00098 .00020 .00000 .00000 .00000
   -87.3  -80.2 -102.2  -80.9 -127.4 -156.4 -129.3 -176.6  720.0  720.0  720.0
```

As can be seen, the M_2-wave has the highest amplitude of 11 nm/s^2. The gravity reduction due to ocean loading can be computed by using these information as

$$\delta g_{oload} = \sum_j f_j \cdot A_{cj} \cdot \cos\left(\omega_j \cdot t + \chi_j + u_j - \Phi_{cj}\right) \qquad 3.5$$

The amplitude and phase angle are denoted as A and Φ, respectively. The parameters f and u depend on the longitude of the lunar node [🕮 7], χ is the astronomical argument of the tide [🕮 8].

Reduction due to Polar Motion

This reduction compensates long-periodic effects due to the deviations of the instantaneous pole from the Conventional International Origin (CIO), i.e. this reduction can be neglected for local relative surveys of some hours. The correction may be computed to

$$\delta g_{pol} = \delta_{pol} \cdot \omega_{Earth}^{2} \cdot R \cdot \sin 2\varphi \cdot (x \cdot \cos\lambda - y \cdot \sin\lambda) \qquad 3.6$$

where d_{pol} is the gravimetric amplitude factor that can be chosen equal to the tidal amplitude factor of 1.16, w is the angular velocity of the earth (2π per day), R is the earth radius of 6,371 km and j, l are the geographical latitude and longitude of the station. Finally, x and y denote the pole coordinates (usually given in arc-seconds) that are provided by the International Earth Rotation Service (IERS).

4. Adjustment

4.1 Methods

There are mainly two conceptual approaches to adjust gravity networks. The first method uses corrected measurements as observations whereas the second method introduces gravity *differences* into the adjustment. This approach has the advantage that the number of additional unknown polynomial drift coefficients highly decreases as the drift effects are reduced significantly by differencing. Therefore, you may only need 0 to 2 coefficients[4] to model the drift, whereas you might need up to 5 coefficients for the first method [📖 5]. The disadvantage is that these derived observations are algebraically correlated with each other and residuals are more difficult to understand as they always affect two stations (problem of separation).

4.2 Observation Equation

The observation equation for the second method, the adjustment via gravity differences, can be expressed as

$$\begin{aligned} \Delta L_{ij} &= (g_j - \delta g) - (g_i - \delta g) - [\Delta F(z_j) - \Delta F(z_i)] + [P(z_j) - P(z_i)] + [d(t_j) - d(t_i)] \\ &= g_j - g_i - [\Delta F(z_j) - \Delta F(z_i)] + [P(z_j) - P(z_i)] + [d(t_j) - d(t_i)] \end{aligned} \qquad 4.1$$

where g is an absolute gravity value, δg is the (unknown) offset of the relative gravity meter which is eliminated by differencing the observables. ΔF stands for the calibration function, P for the periodic error function and d denotes the drift function.

If a linear drift *(D)* is assumed and, additionally, the scale factor *(F)* of the gravity meter is to be determined, equation 4.1 can be written in the form

---

4) Here, no drift compensation at all is necessary for short-time surveys if a gravimeter with favorable behavior is used. For long-time surveys, parabolic drift compensation is usually sufficient.

$$g_j - g_i = \Delta L_{ij} \cdot F + D \cdot \Delta t_{ij} \quad \Leftrightarrow \quad \Delta L_{ij} = \frac{g_j - g_i - D \cdot \Delta t_{ij}}{F} \qquad 4.2$$

The partial differentials of this equation that are to be introduced into the design matrix can be easily derived:

$$\frac{\partial \Delta L_{ij}}{\partial g_i} = -\frac{1}{F} \approx -1 \qquad 4.3$$

$$\frac{\partial \Delta L_{ij}}{\partial g_j} = +\frac{1}{F} \approx +1 \qquad 4.4$$

$$\frac{\partial \Delta L_{ij}}{\partial F} = -\frac{g_j - g_i - D \cdot \Delta t_{ij}}{F^2} \approx g_j - g_i = -\Delta L_{ij} \qquad 4.5$$

$$\frac{\partial \Delta L_{ij}}{\partial D} = -\frac{\Delta t_{ij}}{F} \approx -\Delta t_{ij} \qquad 4.6$$

Note that F is approximately 1.0. Nevertheless, the expressions above clearly show that if you additionally want to determine the scale factor, the equations are no longer linear.

4.3 Least-Squares Adjustment

After having set up the design matrix A, the vector of the shortened observations $l = L - L_0$ (L_0: vector of approximated observations) and the stochastic model with its weight matrix P the adjustment can be computed straightforwardly:

$$\hat{\mathbf{X}} = \mathbf{X} + (\mathbf{A}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{A})^{-1} \cdot (\mathbf{A}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{l}) \qquad 4.7$$

It is useful to test the drift parameter as well as the scale factor statistically to confirm that these additional unknowns are really significant. If this is not the case, they should be canceled from the adjustment.

Moreover, the standard deviation a priori, s_0, and a posteriori, s_0, should be compared. The a posteriori value is computed by

$$s_0^2 = \frac{\mathbf{v}^{\mathbf{T}} \cdot \mathbf{P} \cdot \mathbf{v}}{f} \qquad 4.8$$

where **v** is the vector of the residuals and f is the number of degrees of freedom. If there is an unexpected discrepancy between σ_0 and s_0, the reasons may lie in wrong a-priori assumptions (incorrect stochastic model) or in an inappropriate functional model, e.g. because non-linear calibration terms are neglected although they have a concise impact.

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