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Physical Geodesy

Second,
corrected edition

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This book is dedicated to the memory of

Weikko Aleksanteri Heiskanen

(1895–1971)

Pioneer, initiator, and coauthor of “Physical Geodesy”,
whose dreams have come true in a way unexpected to
all of us.

Foreword

Almost the period of one generation has passed since 1967, the year of the first release of *Physical Geodesy* by Weikko A. Heiskanen and Helmut Moritz. Soon this book became a bestseller. Today, when studying publications dealing with physical geodesy, not surprisingly the book is still frequently quoted. Have the clocks been stopped since then? Not at all, time has flown as fast as usual or maybe even faster – at least in someone’s imagination. However, excellent quality is correlated with a long life expectation. This is the reason why “the book” still plays an important role in geodetic science and beyond.

In the last decades, nevertheless, geodesy has certainly continually developed further – on the one hand by new computational methods and ideas and on the other hand by modern measurement techniques. This is where the story of this book starts.

Several years ago, I tried to convince Helmut Moritz on the necessity of a new edition of *Physical Geodesy*. Even if I encountered some interest, I did not manage to completely succeed. “Steter Tropfen höhlt den Stein” (persistent drops hollow out the stone), I thought and started to repeat my request regularly. The reason for my somehow obstinacy originated from the past. In 1993, I got the chance to support Helmut Moritz in writing the book entitled *Geometry, Relativity, and Geodesy*. For me, this was a tremendously exciting time where we developed a great cooperation in any respect. Immediately after this experience, I manifested my desire of another chance for a cooperation. In these days, the idea of a new edition of *Physical Geodesy* matured.

Finally, the persistent drops succeeded. I cannot tell you the Why and the When; suddenly we had a contract with the Springer Publishing Company. To me it seemed as if the wheel of time had been turned back – thank you, Helmut!

Many persons deserve credit and thanks. Prof. Dr. Klaus-Peter Schwarz, retired from the Department of Geomatics Engineering of the University of Calgary, strongly influenced the balance between keeping, eliminating, updating, and adding topics.

Prof. Dr. Herbert Lichtenegger, retired from the Institute of Navigation and Satellite Geodesy of the Graz University of Technology, was a reviewer of the book. He has critically read and corrected the full volume. His many suggestions and improvements, critical remarks and proposals are gratefully acknowledged.

Prof. Dr. Norbert Kühtreiber from the Institute of Navigation and Satel-

lite Geodesy of the Graz University of Technology has helped with constructive critique and valuable suggestions. Furthermore, he has strongly helped to shape Chap. 11 by providing numerical examples and his valuable experience on the practical aspects of geoid computation.

In several fruitful discussions, Prof. Dr. Roland Pail from the Institute of Navigation and Satellite Geodesy of the Graz University of Technology has provided his rich experience on the space gravity missions. Parts of his structured lecture notes are mirrored in the corresponding section. He also deserves thanks for a careful proofreading of this section.

The cover illustration was designed and produced by Dipl.-Ing. Elmar Wasle of TeleConsult Austria GmbH (www.teleconsult-austria.at). When presenting this illustration to the Springer Publishing Company, the response was extremely positive because of its eye-catcher quality.

The index of the book was produced using a computer program written by Dr. Walter Klostius from the Institute of Geoinformation of the Graz University of Technology. Also, his program helped in the detection of some spelling errors.

The book is compiled with the text system $\text{\LaTeX}2\epsilon$. One of the figures included is also developed with $\text{\LaTeX}2\epsilon$. The remaining figures are drawn by using CorelDRAW 11. Primarily Dr. Klaus Legat from the Institute of Navigation and Satellite Geodesy of the Graz University of Technology deserves the thanks for the figures. He was supported by Prof. Dr. Norbert Kührtreiber. The highly academic level of the producers assures a seal of quality. Many of these figures are redrawings of the originals in Heiskanen and Moritz (1967).

I am also grateful to the Springer Publishing Company for their advice and cooperation.

The inclusion by name of a commercial company or product does not constitute an endorsement by the authors. In principle, such inclusions were avoided whenever possible.

Finally, your ideas for a future edition of this book and your advice are appreciated and encouraged.

The selection of topics is certainly different from the original book written by Heiskanen and Moritz. However, basically we tried not only to keep the overall structure wherever possible but also to leave the text unchanged. The primary selection criteria of the topics were relevancy, tutorial content, and the interest and expertise of the authors. A detailed description of the contents is given in the Preface.

Preface

This book is a university-level introductory textbook. Physical geodesy is the science of the figure of the earth and of its gravity field. Particular emphasis is put on the interaction between geometry, especially GPS, and modern gravitational techniques. The mathematical tool is potential theory. More about the purpose and application of physical geodesy will be found in the subsequent motivation. For better readability, some repetitions are purposely used. The mathematical apparatus is kept as simple as possible.

The book is divided into 11 chapters, a section of references, and a detailed index which should immediately help in finding certain topics of interest.

The first chapter is an introduction to mathematical potential theory to the extent needed in the present book. More precisely, it is “classical” potential theory as represented, e.g., by the book of Kellogg (1929) (this is our usual mode of quoting references, by name[s] and year). Mathematicians will notice immediately that the presentation, as in most textbooks on theoretical physics, is informal: proofs are frequently omitted or replaced by “heuristic” considerations.

The second chapter introduces the gravity field of the earth, e.g., the force of gravity, level surfaces and plumb lines, the geoid, and coordinates naturally related to them: astronomic latitude and longitude as well as heights above the geoid. A powerful tool are developments in spherical harmonics. The natural reference surface is an ellipsoid of revolution equipped with a “normal” gravity field. This gives us a “Geodetic Reference System” (GRS) or World Geodetic System (WGS). Deviations of the real gravity field quantities from the corresponding reference quantities are small and can be linearized. This makes it possible to treat geodetic problems as relatively simple boundary-value problems of potential theory. A well-known classical solution is Stokes’ integral formula.

The third chapter deals with gravity reductions, in particular reductions using the theory of isostasy. A first link to geophysics is established in this way.

The fourth chapter considers the problem of heights, which is more complicated than one would think at the beginning. The first four chapters are an update of the old book by Heiskanen and Moritz (1967), which serves as the template of this book.

The fifth chapter is central in several respects. It is vastly expanded as compared to the former book and has a completely different structure.

The problem of interrelating geometric and physical aspects is met here in all its complexity, from the global (Part I) to the classical local aspects (Part III), the regional “three-dimensional geodesy” in the pre-satellite sense (Part II) forming a transition. One could also say, Part I is integral and Part III is differential. Part I, geocentric and global reference systems, has been made possible only by highly precise geometric satellite methods. The problem of the third dimension, one of the most difficult tasks of geodesy, is formulated and solved here in the most direct and natural way. Part II is an attempt to solve this problem classically in a nondifferential way, but the weak link is the measurement of the zenith distances which are too inaccurate because of atmospheric refraction. The classical way out of this dilemma, still valid today, is the astrogeodetic integration of deflections of the vertical as discussed in Part III.

The sixth chapter is relatively slight, treating the computation of the gravity field up to about 10 km, with a view to application to airborne gravimetry. It is a streamlined version of the old Chapter 6.

The seventh chapter corresponds to the old Chapter 9, but it is greatly expanded to reflect the enormous progress of satellite methods for the determination of the global gravitational field. The main problem has been the gap between this global field at high elevations and the detailed but ill-distributed terrestrial gravity measurements. The new dedicated satellite missions, intended to close this gap, are described.

The eighth chapter, on Molodensky’s and related theories, is again considerably expanded, owing to their great conceptual importance. Molodensky was the first to base physical geodesy on boundary-value problems on the physical earth’s surface rather than on the geoid. His conceptual frame includes also astrogeodetic methods. Although conceived by Molodensky as a method to avoid the reduction of gravity to sea level, in mountain areas it works best if combined with isostatic and similar gravity reductions which are now familiar as “remove-restore” techniques. A great change with respect to the earlier book is the fact that now we understand Molodensky’s theory much better, so that we can treat it now completely with elementary mathematics, without the use of integral equations.

The ninth chapter is a practically unchanged update of the old Chapter 7. The statistical treatment of gravity has undergone an enormous and unexpected increase in importance, theoretical as well as practical, so as to warrant a special new chapter.

The tenth chapter deals with least-squares collocation. This is a great synthesis of a generalization of least-squares prediction of gravity treated in the ninth chapter, with the theory of Hilbert space with kernel functions, and with ideas of gravity reduction in the theory of Molodensky. This synthesis

is due to a small publication by Krarup (1969). It has an extremely simple mathematical structure and uses only matrix methods well-suited for electronic computation; still, it permits the combination of virtually all geodetic data types. Thus it has become very popular for numerical computation.

The eleventh chapter illustrates these various methods by computations in Austria, which combines a difficult topography with easily available data.

Internet citations within the text omit the part “http://” if the address contains “www”; therefore, “www.esa.int” means “http://www.esa.int”. Usually, internet addresses given in the text are not repeated in the list of references. Therefore, the list of references does not yield a complete picture of the references of which we have been benefiting.

The use of the internet sources caused some troubles for the following reason. When looking for a proper and concise explanation or definition, quite often identical descriptions were found at different locations. So the unsolvable problem arose to figure out the earlier and original source. In these cases, sometimes the decision was made, to avoid a possible conflict of interests, by omitting the citation of the source at all. This means that some phrases or sentences may have been adapted from internet sources. On the other side, as soon as this book is released, it may and will also serve as an input source for several homepages.

For bibliographical references, the most readily accessible or most comprehensive publication of an author on a particular topic is given rather than his first. The list of references does not aim at completeness; some important publications may have been omitted but never on purpose.

The (American) spelling of a word is adopted from Webster’s Dictionary of the English Language (third edition, unabridged). Apart from typical differences like the American “leveling” in contrast to the British “levelling”, this may lead to other divergences when comparing dictionaries. Webster’s Dictionary always combines the negation “non” and the following word without hyphen unless a capital letter follows. Therefore “nongravitational”, “nonpropulsed”, “nonsimultaneity” and “non-European” are corresponding spellings.

Symbols representing a vector or a matrix are in boldface. The inner or scalar product of two vectors is indicated by a dot “.”. The norm of a vector, i.e., its length, is indicated by two double-bars “||”. Vectors not related to matrices are written either as column or as row vectors, whatever is more convenient.

Preface to the second edition

Compared to the original version, this second edition only answers four times the question marks resulting from a wrong label denotation and, for L^AT_EX2 ϵ experts, some “overfull \hbox” warnings indicating too long lines without finding appropriate linebreaks were eliminated. Furthermore, one misspelling was corrected, some punctuation problems were solved differently, and a few sentences were reformulated or updated. Therefore, it is considered a corrected version and not a revised one. This is a significant difference because it implies that possible advice for improvements from readers or reviewers were not taken into consideration. The reason for this is not a haughty disregard of other ideas but the tightness of the time schedule. To date only a very small number of reviews has been released. This almost coincided with the message of the Springer Publishing Company that the first edition will be sold out shortly. Therefore, we are grateful to those who gave us advice and we further encourage all readers accordingly because their support might help to improve another edition.

April 2006

B. Hofmann-Wellenhof H. Moritz

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