Motivation

As we have already indicated in the Preface, the subject of physical geodesy is the study of the gravity field and the figure of the earth. In former times, the scientifically relevant "figure of the earth" was the *geoid*, which is defined as one of the equipotential surfaces of the earth's gravity potential, of which the (mean) surface of the oceans forms a part. So the gravity field immediately enters into the very definition of "figure of the earth". "Heights above sea level" are heights above the geoid, and thus are both physically and geometrically defined.

Gravity, essentially caused by the earth's gravitational attraction, has always been determining the life of humankind, from walking on a hilly road to crossing the oceans by ship or by airplane. It has also formed the shape of our planet.

Scientific geodesy started when leading scientists such as Newton recognized that the earth cannot be a sphere but must rather be flattened because of the earth's rotation. Not very much, but probably measurably. This was one of the greatest scientific problems of that time.

Therefore, around 1740, the French Academy of Sciences undertook two expeditions, one under Bouguer to Peru and one under Maupertuis to Lapland. Their purpose was to measure the length of a meridional arc of, say, 1 degree of latitude, one close to the equator and one close to the North Pole. The difference between the two results is a measure of the flattening, which is the deviation (with respect to the sphere) of the earth ellipsoid. These measurements clearly indicated that the global figure of the earth is an ellipsoid of revolution, at least approximately.

The next century was characterized by attempts to define the figure of the earth more precisely. C.F. Gauss (1777–1855), the "princeps mathematicorum", raised geodesy to the rank of a science. He did this by his theory of surfaces – which finally led to General Relativity, cf. Moritz and Hofmann-Wellenhof (1993) – and his adjustment by least squares, the first of all statistical estimation methods. He liked practical geodetic work and measured a triangulation net. Gauss also introduced the geoid as the "mathematical figure of the earth" defined as a level surface of the gravity field.

The geoid deviates from a well-fitting ellipsoid (e.g., the Geodetic Reference System 1980) by less than 100 meters. Geocentric positions nowadays can be determined by GPS to an accuracy of better than 1 decimeter in a purely geometric way. We may define these positions either in terms of geocentric Cartesian coordinates or as ellipsoidal coordinates φ, λ, h . So the geometry of the earth can be determined largely independently of the gravitational field, thanks to GPS and other satellite techniques. Still, the gravitational field is needed, e.g., for determining the orbits of the satellites themselves.

It is probable that, by the influence of GPS, gravity anomalies Δg will gradually be replaced by gravity disturbances δg . This is taken into account in the present book.

Gravity has become one of the most sought-for and most interrelated data in geophysics, and every increase of accuracy has immediately generated new needs. For instance, the ocean surface as determined by satellite altimetry is not an exact equipotential surface because of small tilts due to ocean currents. Thus, this "ocean topography", measured by comparing the results of satellite altimetry with precise gravimetric geoids determined by combining various methods, provides important boundary conditions for oceanography.

Already Clairaut related the density of the masses inside the earth with internal gravity on the condition of hydrostatic equilibrium, and this question with the classical title "The figure of the earth" can now be reconsidered in the light of satellite data; cf. Moritz (1990).

Geological phenomena in the earth's crust and upper mantle such as isostasy and plate tectonics require an interaction between geodesy, geophysics, and geology.

Polar motion and anomalies in the earth's rotation are largely caused by the ceaseless circulation of the air masses defining weather. Earth rotation is now monitored by laser and GPS, which provides an unexpected link between geodesy and meteorology; cf. Moritz and Mueller (1987).

New measuring techniques related to inertial navigation systems (INS) require an interaction between the geometry and the gravitational field. This has considerable practical consequences, e.g., in tunnel surveying. GPS stops short in front of a tunnel, and INS or conventional surveying methods must take over. Either of them, however, does depend on the gravity field.

The terrestrial measurement of gravity is very time-consuming. Airborne gravimetry has become operational only after the inertial and gravitational forces have become separable by combination with GPS.

Not all of this can be treated in detail in the present introductory book. It is, however, intended as a solid, mathematically oriented and not too difficult treatise on graduate level leading to one's own postgraduate research.

The book by Heiskanen and Moritz (1967) stood at the transition between classical and satellite geodesy. Similarly, the present book stands at the beginning of an era characterized by "sensor integration", data combination, and kinematic and navigational techniques.