Real Orbits

3

Ideal and Real Orbits

THE orbits that we have discussed in the preceding two chapters are often called Keplerian orbits, named after Johannes Kepler, and sometimes referred to as *ideal orbits.* If we imagine a satellite in orbit around the Earth, and the only force acting upon it is that of a perfect inverse square law gravity field (see Chapter 1), then the resulting orbital motion is described as ideal. The main distinguishing feature of an ideal orbit is that the attributes defining the orbit—its shape, size, and orbital inclination (see Chapter 2)—all remain fixed; that is to say, they do not change with time.

In contrast to an ideal orbit, the defining attributes of *real orbits* do change, and this is due to the effects of *orbital perturbations.* This is just a fancy phrase to describe forces that act on the satellite, in addition to the inverse square law of gravity, that cause its path to differ from an ideal circular or elliptical orbit. When the mission analysis team designs the orbital aspects of an Earth orbiting mission, it has to take these aspects into account. There are a number of different sources of perturbations, and this chapter discusses the most common ones.

The gravity field of the Earth is very close to being described by Isaac Newton's inverse square law, although there are small departures from this. The dominant part of Earth's gravity field, described by the inverse square law, is sometimes referred to as the *central gravity field.* The small departures are sometimes called *gravity anomalies* and are a form of perturbation that causes the satellite to deviate from ideal orbital motion. Speaking more generally, the effects of orbit perturbations from all sources are small compared to the central gravity field. As a consequence, real orbits are basically the same shape as ideal orbits—circles and ellipses—but their shape, size, and orbital inclination change slowly over time.

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Earth Orbit Perturbations

For Earth-orbiting spacecraft, there are four main sources of orbit perturbation that can have a significant effect on the orbital motion:

1. Gravity anomalies
2. Third-body forces
3. Aerodynamic forces
4. Solar radiation pressure

In what follows we summarize how these work, and discuss in what way they affect the characteristics of the orbit. There are many other perturbation forces of lesser magnitude that influence the motion; in fact, the list of possible perturbations is long. For a particular spacecraft, the choice of which perturbations to include in controlling and operating the spacecraft depends on how precisely we need to know where the spacecraft is located in space. For example, the position of some current Earth-observation satellites needs to be known to an accuracy of a few centimeters in order to get the maximum amount of useful science from the payload. In this case, a long list of orbit perturbations must be taken into account to determine the position of the spacecraft so accurately. On the other hand, to operate a geostationary orbit communications satellite successfully, it may be necessary to know only that it is located somewhere within a 100-m (328-foot) box, so that the ground station antenna can be pointed to the correct point in the sky. In this case, the use of just three perturbation forces—gravity anomalies, third-body forces, and solar radiation pressure—are quite adequate in achieving this objective.

Finally, before we discuss orbit perturbations, the question arises, why are we interested in this topic? (Readers who are not interested should move on to the next chapter. The rest of the chapters in this book do not require reading this chapter first.) There are two reasons why this topic is of interest. First, an important requirement in the operation of a spacecraft is to know its position in space over time. For example, the operators need to know with confidence that at a specific time tomorrow the spacecraft will rise above the horizon of a ground station, so that it can be commanded to do tasks and it has an opportunity to transmit its payload data to the ground. Clearly, the real perturbed orbit characteristics need to be used in order to make this prediction accurately. Also, in designing low Earth orbit operations, such as the rendezvous of a space shuttle with a space station, orbital perturbations need to be included in the analysis. If they are not, then when you think the two spacecraft have come together in orbit, you will find that they are still tens or even hundreds of kilometers apart! This is especially true if the perturbations due to gravity anomalies in low Earth orbit (LEO) are neglected in the mission planning.

Second, the action of orbit perturbations leads to a requirement for the spacecraft to control its orbit. To understand this, we need to recall the discussion in Chapter 2 about how the best orbit is chosen for a particular spacecraft mission. In summary, the mission orbit is chosen to put the spacecraft into the best place in order that its payload can most effectively achieve the mission's objectives. Having chosen the mission orbit, the spacecraft will then be launched there. Now, if the orbit was ideal, then its shape, size, and orbital inclination would remain fixed, and the spacecraft would continue to reside in its best mission orbit. But we now know that when we launch into the mission orbit, perturbations will change the orbit's attributes over time, which means that the orbit evolves ultimately into one that is inappropriate for the mission objectives. This leads to a requirement to fire small rocket thrusters on the spacecraft to counter the effects of orbit perturbations, so that the spacecraft stays in the mission orbit. This process is referred to as *orbit control*, and is planned and executed routinely by the operators of the spacecraft.

Gravity Anomalies

If the Earth were perfectly spherical, and its internal density distribution had a particularly simple form, then the gravity field of Earth would be a perfect inverse square law, as described by Isaac Newton. However, Earth is not perfectly spherical, nor does it have a simple internal mass distribution. There are topographical features that spoil that perfection, such as mountains that are 8 km (5 miles) high and ocean trenches that are 11 km (7 miles) deep. In addition, Earth's shape is basically that of an oblate spheroid (Fig. 3.1)—a squashed sphere—such that if you were to stand on the poles, you would be 21 km (13 miles) closer to Earth's center than if you stood on the equator.

This is not too surprising given that Earth rotates on its axis once per day, causing the equator to bulge and the poles to flatten. This degree of flattening of Earth—21 km in a radius on the order of 6400 km (4000 miles)—is not too noticeable, however. For example, if you were to stand on the moon and look back at Earth, this degree of oblateness would probably not distract you from the apparent spherical perfection of the brilliant blue orb in the black sky. However, if you think of it in terms of the possible perturbing effects on a spacecraft in a LEO, there is an extra 21 km of crust—of gravitational mass—at the equator, which will have a significant

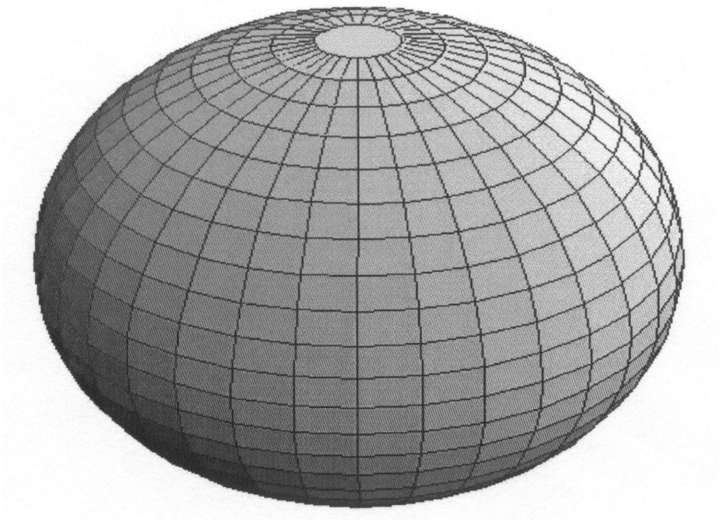


Figure 3.1: The basic shape of Earth is an oblate spheroid, flattened at the poles and bulging at the equator. Note that the actual oblateness of Earth is much less than the shape illustrated!

effect on the spacecraft's motion as it flies over the equatorial region. The gravity field of Earth deviates from that proposed by Newton, causing gravity anomaly orbit perturbations.

What effect does the oblate shape of Earth have on the motion of orbiting satellites? The answer to this is a fairly complex affair, and one that has always challenged me in my career as a teacher in space engineering, even when I am allowed to use mathematics! There are two main effects on the orbit, perigee precession and nodal regression, both of which produce major changes in the orbital motion.

Perigee precession is a gravity anomaly perturbation that affects elliptic orbits. In an ideal elliptic orbit, the major axis of the ellipse, that is, the line from perigee to apogee, remains fixed in direction. If you decided you wanted your spacecraft to be in an orbit with the perigee point above the North Pole, then you would launch into a polar orbit in such a way as to achieve this. In the absence of gravity anomalies, the perigee would remain frozen above the North Pole as required. However, the presence of the extra gravitational mass around the equator, due to Earth's oblateness, causes a greater acceleration on the spacecraft in the perigee region, which in turn causes the trajectory to curve a little more acutely. As a result, when the spacecraft climbs to its apogee, the apogee point has moved, causing the line of the major axis to rotate in the plane of the orbit. This is illustrated in

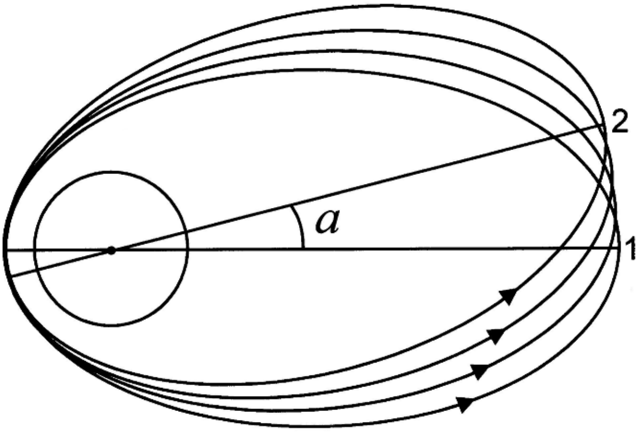


Figure 3.2: Perigee precession causes the major axis of an elliptical orbit to rotate in the plane of the orbit, such that the initial major axis 1 will rotate to major axis 2 after a number of orbit revolutions.

Figure 3.2, where the major axis of the orbit has rotated through an angle *a* (from 1 to 2) over four orbit revolutions.

In fact to move the major axis through this angle may require many orbit revolutions, particularly for the size of orbit shown, but the diagram has exaggerated the effect to aid clarity. Coming back to our orbit with the perigee above the North Pole, due to the effects of perigee precession, the perigee point will not stay fixed above the Pole but will move steadily around the orbit.

The rate at which the perigee moves is dependent on the size, shape, and orbital inclination of the orbit, so it is difficult to generalize. One thing that can be said, however, is that low orbits are affected more than high ones, since the influence of the extra mass due to the equatorial bulge is more strongly felt when a spacecraft makes a low pass over the equator. To give you an idea of the numbers, for a low elliptical orbit with a perigee height of 300 km (185 miles), an apogee height of 500 km (310 miles) and an orbital inclination of 30 degrees, the major axis of the orbit will rotate at a rate of about 11 degrees per day. To get a feel for altitude dependence, if we stick with the same orbital inclination but increase the perigee altitude to 1000 km (620 miles) and the apogee altitude to 10,000 km (6200 miles), then the perigee precesses at a rate of about 2 degrees per day. In terms of magnitude, this is a large perturbation, even for the higher orbit. Other changes to the orbit that mission analysts get excited about, due to other types of perturbation, are typically of the order of fractions of a degree per day!

Nodal regression is a gravity anomaly perturbation that affects both circular and elliptic orbits. The first question is, What is a node? You may remember the answer from Chapter 2. This is just the point on the orbit where the spacecraft crosses the equator. Clearly it does this twice on each orbit revolution, once when traveling from south to north, and once on the other side of the orbit when traveling from north to south. When the spacecraft motion is "ascending" from south to north, the equator crossing is called the *ascending node,* and when descending from north to south we have a *descending node.* The line between the nodes—the intersection of the orbit plane and the equatorial plane—is called the *line of nodes.* For an ideal orbit, the line of nodes remains fixed in direction with respect to the distant stars, but in a real orbit the gravity anomaly perturbation causes it to move around the equator. This nodal movement—or *nodal regression*—is illustrated in Figure 3.3a, and is due to the extra mass associated with Earth's equatorial bulge.

For orbital inclinations less than 90 degrees, the node moves west around the equator (as shown). When the inclination is 90 degrees—a polar orbit—the node remains stationary, and when the inclination is greater than 90 degrees the node moves east. As the node moves, the orbit plane rotates, while the orbital inclination remains constant. This nodal movement, and plane rotation, will continue indefinitely, as shown in Figure 3.3b. The tip of the arrow describes a circle, while the arrow always remains at right angles to the orbit plane. How nodal regression is produced by Earth's oblate shape is a little difficult to explain, but it is related to something called *gyroscopic precession.*

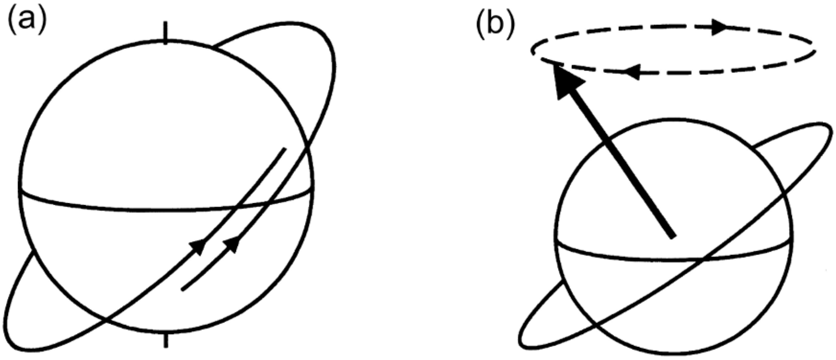
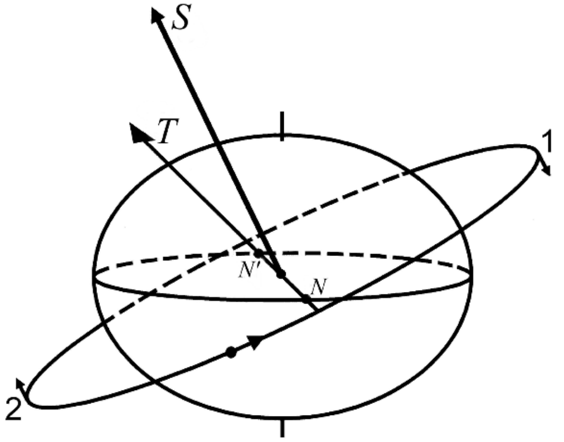


Figure 3.3: (a) In an ideal orbit the node remains stationary, but Earth oblateness causes the node to move so that the spacecraft crosses the equator at a slightly different position on each orbit. (b) Another way of describing the effect is to imagine an arrow that always remains perpendicular to the orbit plane. Over time, Earth's oblateness causes the tip of the arrow to describe a circle.

If you are interested to know what this is about, and perhaps have a bit of a technical background, then read the Nodal Regression box. If not, then you can skip it without compromising your understanding of what follows.

Nodal Regression

The effect of nodal regression on an orbit is similar to the effect of the precession of a gyroscope, when subjected to a torque. A *torque* is a rotational force, like the force you have to apply to remove a bolt from a wheel when you get a flat tire. You apply a force in a rotational sense by pushing at the end of the handle of the wrench. The size of the applied torque is not just to do with the amount of force exerted but also the length of the handle of the wrench you are using. The longer the handle, the greater the "moment arm" and the more torque there is. The axis of the torque is parallel to the direction about which the rotation takes place—in this case the long axis of the bolt.



Now, if we refer to the diagram, Earth is depicted as a rather exaggerated oblate sphere. The orbital motion of the spacecraft about Earth produces a spin axis *S*, called an *orbital angular momentum vector,* which is perpendicular to the orbit plane. At the northernmost position on the orbit, point 1, the gravity force on the spacecraft is deflected slightly downward to the extra mass around the equator, producing a small out-of-plane component of gravity as shown. Similarly, at point 2 a small out-of-plane force is produced, but this time directed upward. The combination of these small forces produces a torque on the orbit shown as the arrow *T*, the direction of which is aligned with the orbit's line of nodes $N - N'$. As with a gyroscope, if the orbit plane is torqued in this way, its spin axis will tend to align itself with the torque axis; that is, the angular momentum vector *S* will precess toward the torque vector *T*. Put more simply, the spin axis *S* will tilt toward the torque axis *T*. Since the spin axis *S* is always perpendicular to the orbit plane, as *S* tilts so does the orbit plane, causing the node *N* to move westward along the equator. Also, given that the torque axis *T* remains parallel to the line of nodes, its direction also rotates westward in the orbit plane. The result is a precessional motion of the orbit spin axis as shown in Figure 3.3b.

How big an effect is nodal regression? If we take a circular orbit typical of a space shuttle, for example 300 km (185 miles) altitude with an orbital inclination of 30 degrees, then the orbit node will move at a rate of about 7 degrees per day in a westward sense. This is again a huge effect compared to other types of perturbation. The effect is less for higher orbits, however, since the spacecraft is further away from the extra mass associated with the equatorial bulge of Earth. For example, a circular orbit at 10,000 km (6200 miles) altitude, with the same inclination, has a modest nodal regression rate of about 0.3 degrees per day.

The bottom line of this discussion about the effects of Earth oblateness is that they are really, really important in low Earth orbit spacecraft operations. If they are neglected, then the position of the spacecraft over time will be in error by many thousands of kilometers!

Gravity Anomaly Perturbations in GEO

Another kind of gravity anomaly perturbation is nicely illustrated by considering the motion of a spacecraft in geostationary Earth orbit (GEO). As we are now aware, the Earth's shape is predominantly that of a squashed sphere, but there is another aspect of the Earth's shape that is surprising. If you were to slice the Earth through the equatorial plane, the shape of the resulting cross section is not circular but approximately elliptical. The Earth's shape can be represented, in an exaggerated way, by the outline shown in Figure 3.4. To get this shape, we take a sphere and give it a good squeeze at the poles to make it oblate, and then give it a small squeeze at the equator, to make the equatorial cross section slightly elliptical. We end up with a form defined by three perpendicular axes, *a*, *b*, and *c*, each having different lengths. From our previous discussions, we know that the equatorial radius *b* is greater than the polar radius *a* by about 21 km (13 miles). But now we are suggesting that *b* and

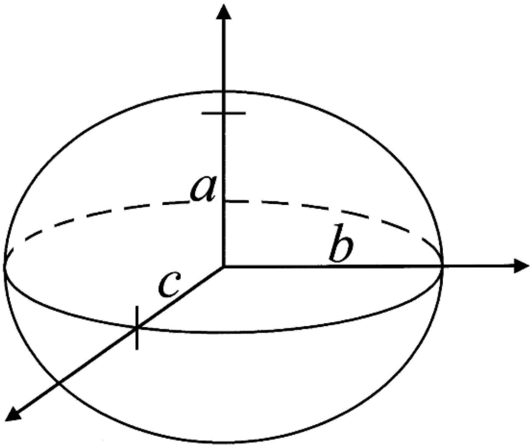


Figure 3.4: The shape of Earth can be approximated by a shape spanned by three axes, all of different length. As well as being oblate, Earth also has an equatorial cross section that is slightly elliptical.

c are different also, but this time by a small amount—less than a kilometer. If we recall that the equatorial radius of Earth $b = 6378$ km (3963 miles), then this difference between *b* and *c* means that the degree of ellipticity of the equatorial cross section is small indeed. However, we now focus attention on this, and discuss the effect it has on the motion of a GEO spacecraft.

But how can such a small deviation in the shape of the equatorial cross section have any effect on an orbiting spacecraft? The answer to this question comes from the fact that although the perturbing forces are small, they tend to act on the spacecraft in the same way on each orbit revolution, so that lots of small changes accumulate to cause an effect that is sizable.

To explain this, in Figure 3.5 we are looking down on Earth and the GEO. The elliptical cross-section of the equator is shown in a rather exaggerated way, with the Greenwich Meridian drawn in to relate the ellipse to Earth's geography. In Figure 3.5a, the points A and B represent the "bulges" in the equatorial cross section that occur at around 160 and 350 degrees longitude east, respectively (it may be helpful to have a globe of the world handy while reading this chapter to help with the geography). This places them in the region of the western Pacific Ocean on the one hand, and the west African coast on the other. A spacecraft is also shown in GEO at an arbitrary position, which happens to be at a longitude of about 40 degrees east. Note that the longitudinal position of the GEO spacecraft would depend on what region it serves; in this case it just happens to be stationed above east Africa and the Middle East.

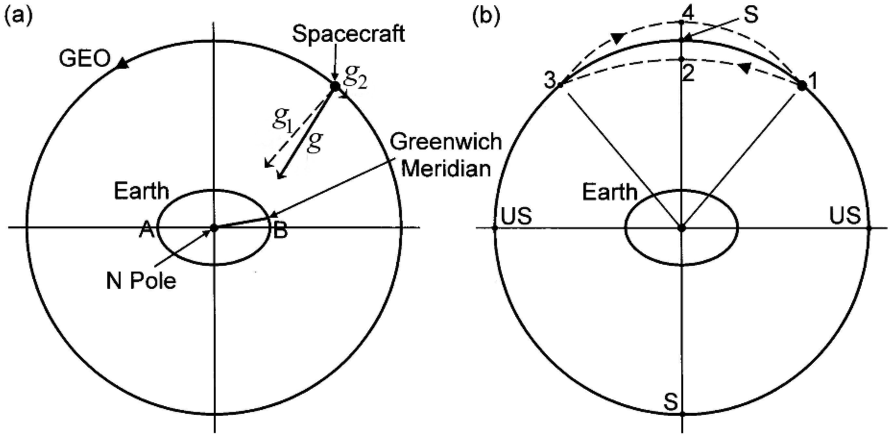


Figure 3.5: (a) Looking down on the elliptical equator of Earth and the GEO. (b) The oscillatory motion of a GEO spacecraft due to the gravity anomaly perturbations caused by the elliptical equator.

An important thing to note about Figure 3.5a is that, for an ideal GEO, the geometry is unchanging. What I mean is that the Earth rotates once per day, in the same time as the spacecraft takes to complete one orbit. The situation is equivalent to rotating the whole of Figure 3.5a about the North Pole axis once per day. In this process the relative positions of Earth and spacecraft do not change. If we now focus on the spacecraft, the gravity force acting on it is modified by the elliptical mass distribution around Earth's equator. Because the spacecraft is closer to the "bulge" at B than that at A, the direction of the gravity force, indicated by the arrow labeled g, is deflected slightly toward B, rather than pointing precisely at Earth's center. The deflection is exaggerated in the diagram for clarity; in reality it is tiny. The resulting force acting on the spacecraft can now be resolved into two *components*, the main one labeled g_1 directed toward Earth's center, and a tiny force component g_2 acting in the local horizontal direction at the spacecraft.

To illustrate the concept of resolving forces into components, a good example is the use of a heavy roller to flatten the bumps in a lawn. If we look at Figure 3.6, there are basically two ways of doing this: we can either push the roller or pull it. Does it make any difference which way we choose? Well, if we resolve the forces into components as shown, we can see that it does. When we push the roller (Fig. 3.6a), the force we use (the solid arrow) can be resolved into two force components (the broken arrows)—one directed horizontally to move the roller along, and another vertical component directed downward. When we pull the roller (Fig. 3.6b), there is the same horizontal component, but now the vertical component is directed upward.

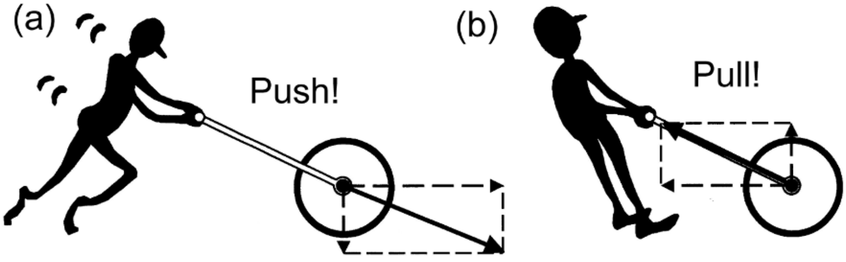


Figure 3.6: The force exerted on a lawn roller (solid arrow) is resolved into horizontal and vertical force components (broken arrows).

If we push the roller, we produce a vertical force component that tends to increase its effective weight and therefore increase friction with the ground, making it harder to move. On the other hand, if we pull the roller, we tend to reduce the ground friction, making it easier to move. The trick of resolving forces into components is often used by engineers, and in the case of the force of gravity acting on a GEO spacecraft, it is just convenient for the argument to resolve it into the vertical and horizontal directions.

Equipped with this intuitive notion of force components, let's return to our discussion about GEO perturbations. Having components of gravity acting in the local horizontal direction is something that we are not very familiar with! In our normal experience, gravity forces always act down the local vertical direction. But in this case the elliptical mass distribution at the equator is producing this rather strange occurrence. The consequences of this for the spacecraft motion are significant. In Figure 3.5b, at point 1 the small horizontal component of gravity force acts in a direction that is opposed to the spacecraft's motion around the GEO; this direction is referred to as *retrograde.* This small retrograde force causes a decrease in the energy of the orbit, with a corresponding decrease in orbit height. What happens if the height of a GEO decreases? The spacecraft orbit speed increases, and it goes round the orbit in slightly less than 1 day; it is no longer synchronous with Earth's rotation. As well as losing height, it also drifts away from its on-station longitude. This situation is depicted by the broken curve from point 1 to point 2. It should be noted that the height change illustrated at point 2 is again exaggerated to make it clear.

At point 2, the spacecraft is at the same distance from the bulges at A and B, and all the gravity force is now directed toward Earth's center. However, because the orbit height is still low, the spacecraft continues to drift, and once it passes point 2 the small horizontal gravity force recurs. But now its direction is reversed, as the spacecraft is now closer to bulge A. The small horizontal gravity force is now in the prograde direction—in the same direction as the spacecraft's motion. This tends to increase the orbit energy, causing the height to increase again, as indicated by the broken curve from point 2 to point 3. At point 3, the spacecraft has regained GEO height, and so becomes synchronous again, halting the drift in longitude. However, the force continues to act in a prograde direction, causing the orbit height to increase above the GEO. Now the orbit speed becomes less than the GEO speed, and the synchronism is again lost as the spacecraft drifts in the opposite direction, represented by the broken curve from point 3 to point 4. Beyond point 4, the force reverses, acting in a retrograde direction once again. This causes a reduction in orbit height until the spacecraft finally returns to its on-station position at point 1.

This rather interesting circuitous journey takes a typical spacecraft quite a long time—of the order of hundreds of days—and is generally a bit of a nuisance for the spacecraft operators, who would prefer the spacecraft to stay at the required on-station position! The operators have to plan and execute orbit control maneuvers to combat the effects of these gravity anomaly perturbations. This means firing small rocket thrusters on the spacecraft (see Chapter 9) to ensure that the spacecraft stays in position. Without this, the spacecraft would oscillate indefinitely in longitude about the stable point S, which is where the minor axis of the equatorial ellipse cuts the GEO arc. In the example above, this would mean that an uncontrolled spacecraft would wander off from its on-station position at 40 degrees to around 120 degrees longitude east (East Africa to Indonesia), and back again on a regular basis. I always find it amazing how such a small variation in Earth's shape can cause such a large change in the spacecraft's orbit!

Note that the orbital positions above the bulges in the equator are unstable—labeled US in Figure 3.5b. Uncontrolled spacecraft positioned near these would move off toward the nearest stable point, labeled S.

Third-Body Forces

Third-body force perturbations are caused by the gravitational influence of a third body in addition to the spacecraft and the Earth. The Earth is not isolated in the universe; there are other celestial bodies out there, the gravitational fields of which can have a significant effect on the motion of an Earth-orbiting spacecraft. The Sun and the Moon have the greatest perturbing effect. If we think about these bodies, and look at Figure 3.7, then the total gravity force governing the motion of the spacecraft becomes a

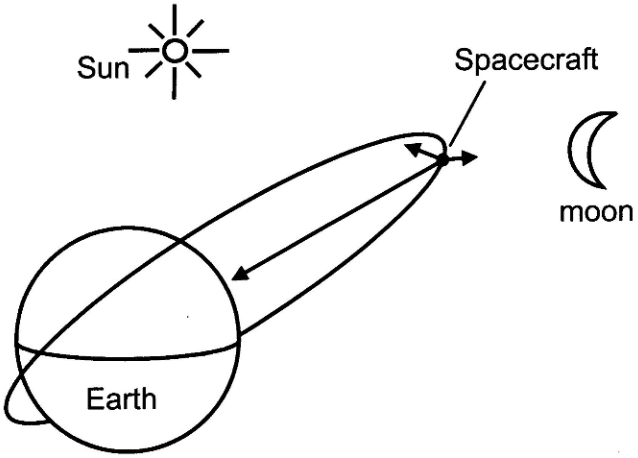


Figure 3.7: The motion of the orbiting spacecraft is influenced not only by Earth's gravity but also by the gravity fields of third bodies, principally the Sun and the moon.

sum of the gravity forces due to the Earth, the Sun, and the Moon. In most applications, the analysis of third-body perturbations includes only the influence of the Sun and the Moon, and then the effects are often called *luni-solar perturbations*. Of course, if high positioning accuracy is required for a particular spacecraft mission, then other third bodies, such as Mars, Venus, and Jupiter, can be included in the analysis until the required degree of precision is achieved.

The effects of third-body forces on low-altitude circular orbits are small. In this case, the Earth's gravity field dominates the contributions from other celestial bodies due to the spacecraft's closeness to the Earth. However, if the spacecraft's orbit takes it to a significant distance from the Earth, for example, in geostationary Earth orbit or at the apogee of a highly eccentric orbit, then the gravitational influence of third bodies is more important. The Earth's gravity force on the spacecraft is reduced because of the greater distance from the Earth's center, whereas the gravitational force due to, say, the Sun has not changed significantly. The overall effect on the orbit remains small, but the ratio of the Sun's gravity force to that of Earth has increased. Therefore, the effects of third-body perturbations are greater in high-altitude orbits. The next obvious question is, What changes do third-body forces produce in the spacecraft's orbit?

In answering this question, an important thing to note is that the gravity field of third bodies generally produce perturbing forces that are out of the plane of the spacecraft's orbit. This is because bodies like the Sun and the

Moon are not often to be found within the plane of the spacecraft's orbit. As a consequence, the main effect of the third-body perturbations is to cause small changes in the plane of the orbit, that is, changes in the orbital inclination. In addition, small oscillations in the size and shape of the orbit are also produced, which can be important for a highly eccentric orbit with a low perigee altitude. In this case, the size and shape variations result in an oscillation in the perigee height, and this can cause the perigee to dip in and out of Earth's atmosphere (see next section on spacecraft aerodynamics). A reentry of the spacecraft into the atmosphere may then result, with the unpleasant prospect of a premature end to the spacecraft's mission life.

Aerodynamic Forces

It seems strange to be talking about aerodynamic effects on spacecraft, because space is a vacuum. Right? Well, as far as people are concerned, as living and breathing creatures, space is *effectively* a vacuum; if you stepped out of a space station in orbit and took off your helmet, then the consequences would reinforce this notion. However, for an orbiting spacecraft the effects of air drag are encountered at heights up to around 1000 km. At these altitudes, there is an atmosphere, but it is extremely thin. To describe how thin it is, let's think about the atmospheric density at an altitude of, say, 800 km (500 miles) at mean solar activity (for a discussion on the effects of solar activity on atmospheric density, see Chapter 6). In every cubic meter of volume at this height there is a mass of air of around 0.000 000 000 000 01 kg, which explains why you can't breath it! Compare this with the density of air at sea level, which is around 1.2 kg/m^3.

The next question is, How can such a thin atmosphere produce aerodynamic effects on spacecraft that perturb their orbital motion (see also the discussion about drag forces on launchers in Chapter 5)? The key to this is to realize that the aerodynamic force on an object is dependent not only on the air density but also on how fast the object is moving through the air. For example, we know that sufficient aerodynamic force can be exerted on a garden fence to knock it down in a winter storm, provided the wind speed is high enough. This force, known as *dynamic pressure*, actually depends on the square of the wind speed. If the wind speed doubles, the force on the fence increases by a factor of four (2^2), if it trebles the force is nine times as big (3^2), and so on. No wonder storm-force winds make short work of fences!

Taking the argument a little higher than a garden fence, we can think about airplanes moving about in the atmosphere. They encounter high-speed winds, but this time the wind is produced by their own motion through the air. The flight of an airplane through the air is resisted by an aerodynamic drag force. This force is measured in a number of different units, but the most common is the Newton, named after Isaac Newton. A Newton of force has a formal definition: it is the force required to accelerate a 1 kg mass by 1 m/sec^2. As we explained in Chapter 1, this is an increase in speed of 1 m/sec for every second that the force is applied. Another perhaps easier way to get a feeling for a Newton of force is to adopt an approximate and informal definition, which is that it is about the weight of a (smallish) apple! Returning to our airplane, and thinking about a civil airliner at cruising altitude, the level of dynamic pressure acting on it due to its motion through the air is on the order of 10,000 to 15,000 Newtons for every square meter of area presented to the air flow. A metric tonne weight is about equal to about 10,000 Newtons, so that's quite a lot of aerodynamic drag, which of course needs to be overcome by the thrust from the jet engines to keep the airplane in the air.

Finally, climbing even higher to orbital altitude, the same principle applies to spacecraft. They encounter a level of aerodynamic drag force that is much smaller than that of an airplane, but it is nevertheless tangible because, although the air density at orbit height is small, the speed of the spacecraft through the atmosphere is high. For each square meter of spacecraft area presented to the air flow, the level of aerodynamic force varies from about 1/100th of a Newton at a 200-km altitude to a tiny 0.000 000 05 of a Newton at a 1000-km altitude. These small forces are produced by the molecules of atmosphere impacting on the spacecraft. The forces seem too small to be of any consequence, but the point is that they act all the time, in a retrograde direction, that is, in a direction opposed to the motion of the spacecraft, producing a small but steady decrease in the orbit height day after day. For example, if the spacecraft resides in a 200-km-altitude circular orbit, this steady decay will lead to the spacecraft's reentering the atmosphere within a short period of time—a few days to a few weeks depending on the characteristics of the vehicle.

We can now begin to understand how aerodynamic drag perturbations change a typical satellite's orbit. The main effects are to reduce the size of the orbit and to change the orbit's shape, making it more circular. To see this, we imagine a spacecraft in an eccentric orbit, as shown in Figure 3.8a, with an initial apogee at point 1, and a perigee height low enough to allow the spacecraft to dip into Earth's thin upper atmosphere. During each perigee passage, the aerodynamic drag forces take energy out of the orbit, so that it does not quite reach the same height on each subsequent apogee (points 2, 3, and so on). In the figure we see two things happening: the size of the orbit is

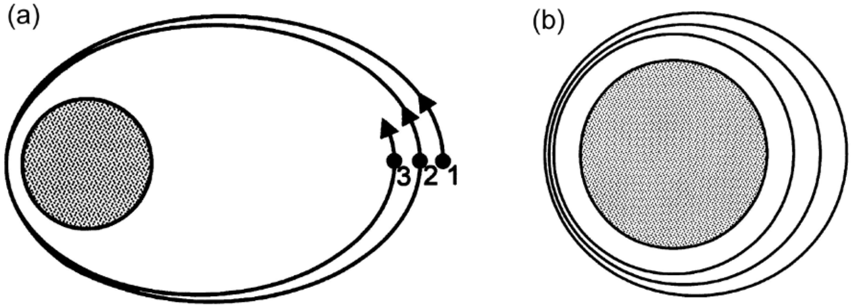


Figure 3.8: (a) The drag effects occur around the perigee of the orbit, mainly causing a change in apogee height. As a consequence, both the orbit size and eccentricity decrease. (b) The evolution of a moderate eccentricity orbit due to air drag, starting at the outer ellipse and ending at the inner circular orbit.

decreasing, and it is becoming more circular. It should be noted that the figure is not drawn to scale, and the sort of decrease in apogee height illustrated would not occur in two orbit revolutions, but in fact would require perhaps hundreds of orbits revolutions. Figure 3.8b shows a similar orbit evolution for an orbit of moderate eccentricity. Again, the major effect is a decrease in apogee height, with a smaller lowering of perigee altitude.

For a spacecraft in a circular orbit, the drag force causes the orbit height to decrease on each orbit revolution, producing a kind of spiral trajectory of diminishing altitude. By how much the orbit height changes depends on the characteristics of the spacecraft. For example, if it is small and massive like the cannonball shot from Newton's cannon in Chapter 2, then the drag effects are relatively small. On the other hand, if the spacecraft is large and of low mass, like a balloon satellite, then the drag effect on orbit height is large. A number of such balloon satellites were launched in the 1960s to bounce radio waves off in early space communications experiments, but their orbit lifetime was generally short due to their vulnerability to drag perturbations. In "ballpark" numbers, for a typical spacecraft the height change per orbit at a 800-km altitude may be on the order of a few centimeters or a few tens of centimeters, whereas at 200-km altitude the height can change by as much as a kilometer or more for each orbit revolution. Clearly, spacecraft in low-altitude circular orbits do not stay in orbit for long.

Another curious feature about drag force on a spacecraft in a low circular orbit is that, although the force acts in the direction opposed to its motion, it actually causes the spacecraft's speed to increase. This is something we do not experience often, and it seems quite counterintuitive. However, a little thought resolves the puzzle. Any force acting in a retrograde sense on a spacecraft will take energy out of the orbit and cause the orbit height to decrease. And as we saw in Chapter 2, the lower the orbit, the faster the spacecraft moves. What's happening here is that, given the decrease in height on each orbit revolution, the spacecraft is actually flying "downhill." The situation is entirely analogous to riding a bicycle down a slope. Gravity pulls it forward and tends to increase its speed, but at the same time we can feel the wind on our faces, which produces a drag force that acts against the motion, tending to slow us down. In the case of the satellite, the gravity pulling it forward is larger than the drag force slowing it down, resulting in the net increase in orbital speed as its orbit height decreases. Surprisingly, I have seen statements in professional journals along the lines of "the aerodynamic drag force decreases the orbit velocity," so you can see that sometimes even the experts get it wrong!

Solar Radiation Pressure

The final orbit perturbation on our brief list of the most important effects causing changes to orbits is solar radiation pressure (SRP). Like drag, the change is produced by a pressure acting on the spacecraft, but this time the pressure is produced by light, in particular the bright sunshine illuminating the spacecraft. Twentieth century physicists were a clever lot, and they first worked out that light reflecting from a surface exerts a pressure on it. As you read this book, the source of light you are using is producing a small force on the page. The fact that nobody noticed this before the 20th century, suggests that light pressure is tiny, and this is indeed the case. You may recall that the drag force is generated by the impact of air molecules on the spacecraft as it speeds through the atmosphere. SRP shares the same mechanism, but the atmospheric particles are replaced in this case by the stream of particles of light—referred to as *photons*—emanating from the Sun.

The magnitude of the pressure is unimpressive, amounting to a few millionth of a Newton for every square meter of spacecraft area presented to the Sun. Comparing this with aerodynamic drag, we find that the magnitudes of drag and SRP effects are about the same for a spacecraft in a circular orbit at around a 600- to 700-km (370- to 435-mile) altitude (depending on the level of solar activity; see Chapter 6). There are differences, however; the magnitude of SRP decreases with the distance from the Sun, as opposed to drag, which decreases with increased height above the Earth. Also, the force of SRP generally pushes the spacecraft away from the Sun, whereas the drag force always acts in a retrograde direction with respect to the spacecraft's forward motion.

It is difficult to summarize the effects that SRP have upon the spacecraft orbit in any meaningful way. Below the 600- to 700-km altitude mentioned above, the SRP perturbations are completely swamped by aerodynamic drag. Above this height, the changes they produce are greatly dependent on the aspect that the orbit plane presents to the Sun. The other thing to remember is that the force is tiny. Generally, small cyclic variations in the size, shape, and orbital inclination are produced. But, as we saw with drag, a tiny force acting on the spacecraft in the same way on each orbit over time can accumulate significant orbit changes. Furthermore, if the spacecraft presents a large area to the Sun—for example, solar panels to convert sunlight into usable electrical power—then the perturbing effects on the orbit are further amplified.

One case where the perturbing effects of SRP can be seen to build up, and explained in a fairly intuitive manner, is that of the motion of a satellite in a geostationary Earth orbit (GEO). This is shown as the circle drawn with a continuous line in Figure 3.9a, seen from the perspective of someone looking down from above the Earth's North Pole.

When the spacecraft is at point 1 in the GEO, the SRP force acts in the direction opposing the motion, causing a small decrease in orbital energy. As a consequence, the orbit height achieved on the opposite side of Earth is reduced, thus forming a perigee at point 2. At this point, the SRP force pushes the spacecraft along, tending to produce a small increase in energy that takes the spacecraft to a higher altitude at point 3. The combination of these effects transforms the circular GEO into the elliptic orbit illustrated in Figure 3.9b, which has its major axis aligned at right angles to the direction of the sunlight. As usual, the discussion has been somewhat simplified; for example, the typical eccentricity produced by SRP perturbations in a GEO is generally much less than that shown in Figure 3.9b. Also, it takes many orbit revolutions for the perturbation to build up this moderate eccentricity, rather than the one revolution discussed above. But the message is clear: SRP perturbations increase the eccentricity of a GEO from an initially circular state to an elliptical one. Why is this important?

If you recall the discussion of the GEO orbit in Chapter 2, its main advantage is that a spacecraft in GEO remains stationary with respect to a ground-based observer, so that communications dishes do not have to move to maintain a link. But this is only true if the orbit is circular, when the spacecraft's speed remains constant. If the orbit becomes slightly elliptical, due to the effects of SRP, then the spacecraft moves a little faster than circular orbit speed at the perigee point of the orbit, and a little slower at the apogee point. From the perspective of someone at the ground station, the spacecraft no longer remains stationary at the point where the commu-

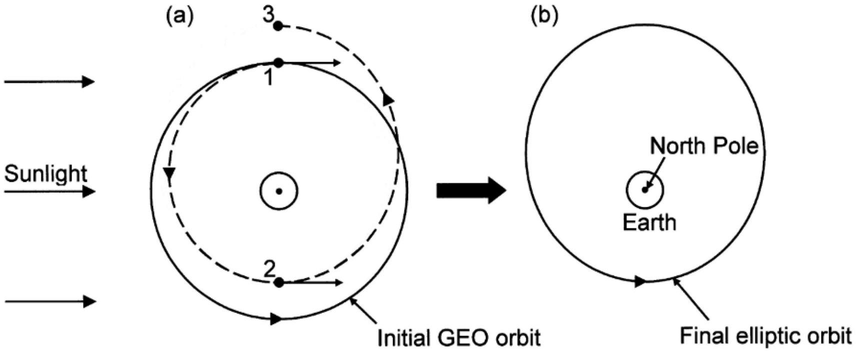


Figure 3.9: The solar radiation pressure perturbation produces an eccentricity in an initially circular orbit.

nications dish is directed, but it appears to wander back and forth through this point, with a period of 24 hours. To counter this, the spacecraft again needs to perform orbit control maneuvers to prevent the SRP perturbations building up. The operators of the spacecraft command it to fire small rocket engines (thrusters) to keep the orbit circular. Although light pressure does seem a strange idea, its reality is confirmed by the fact that spacecraft engineers have to estimate an amount of thruster fuel to put in the spacecraft's tanks in order to control its effect on the spacecraft.

Summary

As can be seen from the above discussion, the topic of orbit perturbations is a fairly complex one. It has been my aim to demonstrate that perturbations need to be taken into account when doing the mission analysis for a real spacecraft project. Another aspect that is implicit in the discussion is that dealing with the perturbation effects requires a good deal of mathematical and computational expertise, which is a routine part of any spacecraft mission analysis activity.

The differences between the ideal, Keplerian orbits of Chapter 2, and the real orbits discussed in this chapter are nicely summed up by the geostationary Earth orbit (GEO). For example, if we have a communications satellite in GEO, then in the absence of perturbations we simply launch the spacecraft into a circular, equatorial orbit at the right height to give an orbit period of one day (see Chapter 2). To someone on the ground, the spacecraft then appears to remain stationary in the sky, and the various communications dishes on the ground that wish to use the satellite simply stare at this fixed position. However, the situation is a little more arduous for the satellite operators in the real world when the effects of perturbations have to be countered. As we have seen, there are three main perturbations that affect a GEO satellite: gravity anomalies, luni-solar perturbations, and solar radiation pressure. Each of these has a distinctive signature with respect to the motion of the satellite as seen from the ground. Gravity anomalies cause the satellite to move away slowly in an east–west (or longitudinal) direction from the point in the sky to which the ground station dish is directed, and in some cases this can cause the spacecraft to disappear over the horizon! Luni-solar perturbations cause changes in the orbital inclination, which in turn cause the satellite's position to oscillate in a north–south (or latitudinal) sense with a period of 1 day. Finally, we have seen that solar radiation pressure effects produce an eccentricity in the orbit, which leads to an east–west (or longitudinal) oscillation as well. Keeping the spacecraft in the line of sight of the ground dish is a nontrivial orbit-control exercise.

The rest of the chapters in this book are less technically challenging than this chapter. With this basic background in orbits, we now move on to Chapter 4 to look at some mission orbits that are a little more exotic than the popular operational orbits that we have already looked at.