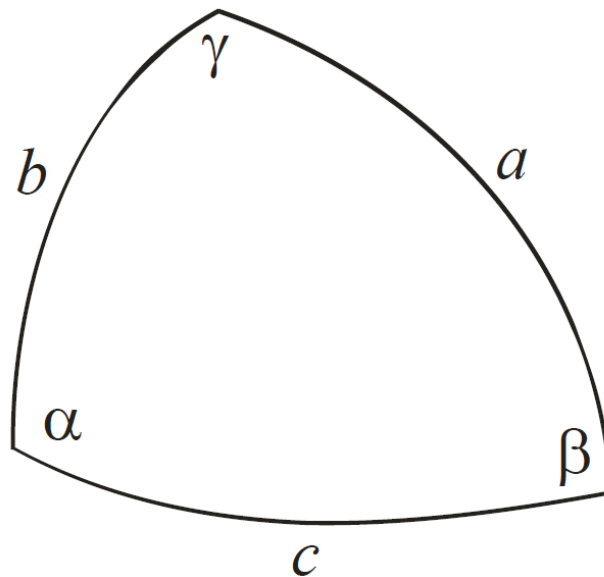


VAJA 9 – SFERNA TRIGONOMETRIJA

1 SPLOŠNI SFERNI TRIKOTNIK



Kosinusni izrek za stranice

$$\begin{aligned}\cos a &= \cos b \cos c + \sin b \sin c \cos \alpha \\ \cos b &= \cos a \cos c + \sin a \sin c \cos \beta \\ \cos c &= \cos a \cos b + \sin a \sin b \cos \gamma\end{aligned}$$

Kosinusni izrek za kote

$$\begin{aligned}\cos \alpha &= -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a \\ \cos \beta &= -\cos \alpha \cos \gamma + \sin \alpha \sin \gamma \cos b \\ \cos \gamma &= -\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c\end{aligned}$$

Sinusni izrek

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}$$

Kotangensni izrek

$$\begin{aligned}\cot a \sin b &= \cos b \cos \gamma + \cot \alpha \sin \gamma \\ \cot b \sin c &= \cos c \cos \alpha + \cot \beta \sin \alpha \\ \cot c \sin b &= \cos b \cos \alpha + \cot \gamma \sin \alpha \\ \cot a \sin c &= \cos c \cos \beta + \cot \alpha \sin \beta \\ \cot b \sin a &= \cos a \cos \gamma + \cot \beta \sin \gamma \\ \cot c \sin a &= \cos a \cos \beta + \cot \gamma \sin \beta\end{aligned}$$

Napierjeve enačbe (analogije)

$$\tan \frac{\alpha + \beta}{2} = \frac{\cos \frac{a - b}{2}}{\cos \frac{a + b}{2}} \cot \frac{\gamma}{2}$$

$$\tan \frac{\alpha - \beta}{2} = \frac{\sin \frac{a - b}{2}}{\sin \frac{a + b}{2}} \cot \frac{\gamma}{2}$$

$$\tan \frac{\alpha + \gamma}{2} = \frac{\cos \frac{a - c}{2}}{\cos \frac{a + c}{2}} \cot \frac{\beta}{2}$$

$$\tan \frac{\alpha - \gamma}{2} = \frac{\sin \frac{a - c}{2}}{\sin \frac{a + c}{2}} \cot \frac{\beta}{2}$$

$$\tan \frac{\beta + \gamma}{2} = \frac{\cos \frac{b - c}{2}}{\cos \frac{b + c}{2}} \cot \frac{\alpha}{2}$$

$$\tan \frac{\beta - \gamma}{2} = \frac{\sin \frac{b - c}{2}}{\sin \frac{b + c}{2}} \cot \frac{\alpha}{2}$$

$$\tan \frac{a + b}{2} = \frac{\cos \frac{\alpha - \beta}{2}}{\cos \frac{\alpha + \beta}{2}} \tan \frac{c}{2}$$

$$\tan \frac{a - b}{2} = \frac{\sin \frac{\alpha - \beta}{2}}{\sin \frac{\alpha + \beta}{2}} \tan \frac{c}{2}$$

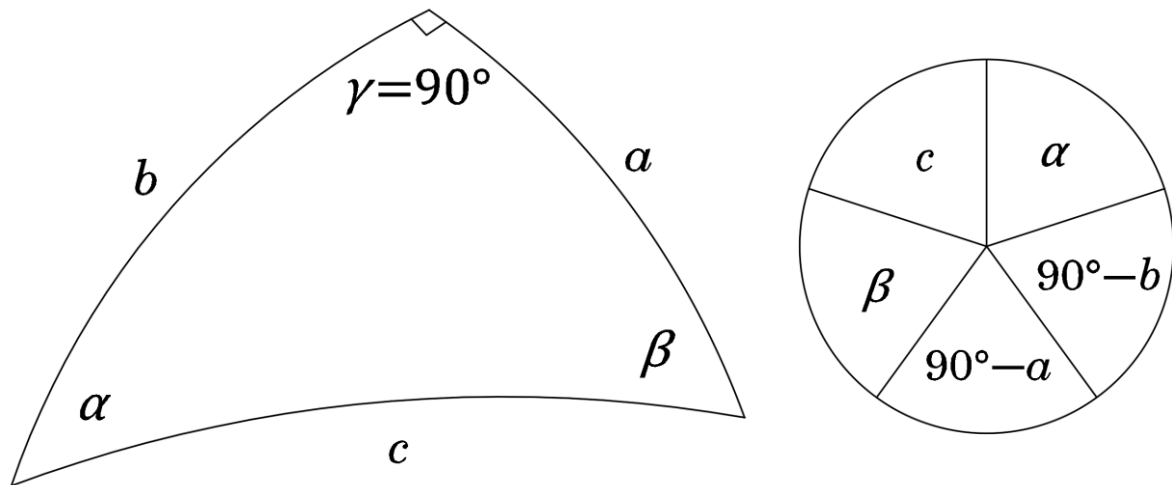
$$\tan \frac{a + c}{2} = \frac{\cos \frac{\alpha - \gamma}{2}}{\cos \frac{\alpha + \gamma}{2}} \tan \frac{b}{2}$$

$$\tan \frac{a - c}{2} = \frac{\sin \frac{\alpha - \gamma}{2}}{\sin \frac{\alpha + \gamma}{2}} \tan \frac{b}{2}$$

$$\tan \frac{b + c}{2} = \frac{\cos \frac{\beta - \gamma}{2}}{\cos \frac{\beta + \gamma}{2}} \tan \frac{a}{2}$$

$$\tan \frac{b - c}{2} = \frac{\sin \frac{\beta - \gamma}{2}}{\sin \frac{\beta + \gamma}{2}} \tan \frac{a}{2}$$

2 PRAVOKOTNI SFERNI TRIKOTNIK



Napierjevo pravilo

Kosinus izbranega elementa v "krogu" (shema desno) je enak (shema zgoraj desno):

- i) produktu kotangensov sosednjih elementov,
- ii) produktu sinusov nasprotnih elementov.

Osnovna oblika izraza

$$\cos c = \cot \beta \cot \alpha$$

$$\cos c = \sin(90^\circ - a) \sin(90^\circ - b)$$

$$\cos \alpha = \cot c \cot(90^\circ - b)$$

$$\cos \alpha = \sin \beta \sin(90^\circ - a)$$

$$\cos(90^\circ - b) = \cot \alpha \cot(90^\circ - a)$$

$$\cos(90^\circ - b) = \sin c \sin \beta$$

$$\cos(90^\circ - a) = \cot(90^\circ - b) \cot \beta$$

$$\cos(90^\circ - a) = \sin \alpha \sin c$$

$$\cos \beta = \cot(90^\circ - a) \cot c$$

$$\cos \beta = \sin(90^\circ - b) \sin \alpha$$

Poenostavljena oblika izraza

$$\cos c = \cot \beta \cot \alpha$$

$$\cos c = \cos a \cos b$$

$$\cos \alpha = \cot c \tan b$$

$$\cos \alpha = \sin \beta \cos a$$

$$\sin b = \cot \alpha \tan a$$

$$\sin b = \sin c \sin \beta$$

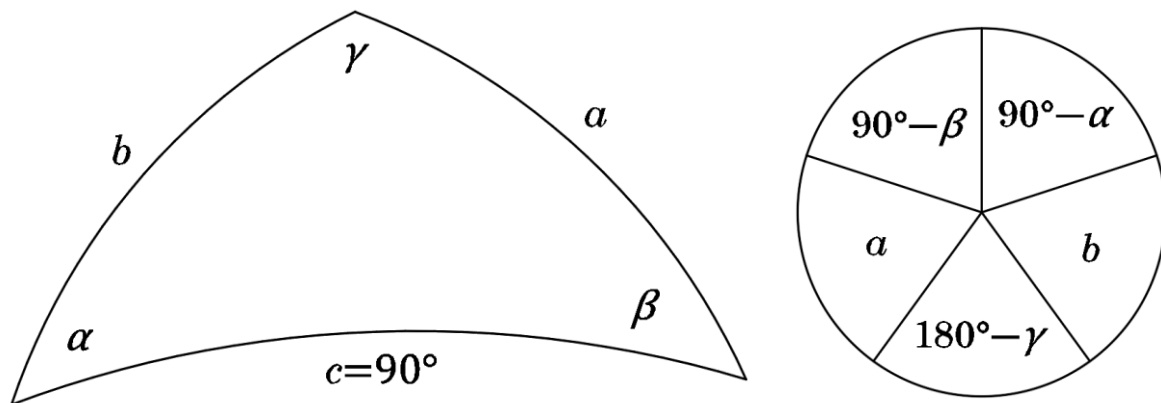
$$\sin a = \tan b \cot \beta$$

$$\sin a = \sin \alpha \sin c$$

$$\cos \beta = \tan a \cot c$$

$$\cos \beta = \cos b \sin \alpha$$

3 PRAVOSTRANIČNI SFERNI TRIKOTNIK



Napierjevo pravilo

Kosinus izbranega elementa v "krogu" (shema desno) je enak (shema zgoraj desno):

- i) produktu kotangensov sosednjih elementov,
- ii) produktu sinusov nasprotnih elementov.

Osnovna oblika izraza

$$\begin{aligned} \cos(90^\circ - \alpha) &= \cot(90^\circ - \beta) \cot b \\ \cos(90^\circ - \alpha) &= \sin a \sin(180^\circ - \gamma) \\ \cos b &= \cot(90^\circ - \alpha) \cot(180^\circ - \gamma) \\ \cos b &= \sin(90^\circ - \beta) \sin a \\ \cos(180^\circ - \gamma) &= \cot b \cot a \\ \cos(180^\circ - \gamma) &= \sin(90^\circ - \alpha) \sin(90^\circ - \beta) \\ \cos a &= \cot(180^\circ - \gamma) \cot(90^\circ - \beta) \\ \cos a &= \sin b \sin(90^\circ - \alpha) \\ \cos(90^\circ - \beta) &= \cot a \cot(90^\circ - \alpha) \\ \cos(90^\circ - \beta) &= \sin(180^\circ - \gamma) \sin b \end{aligned}$$

Poenostavljena oblika izraza

$$\begin{aligned} \sin \alpha &= \tan \beta \cot b \\ \sin \alpha &= \sin a \sin \gamma \\ \cos b &= -\tan \alpha \cot \gamma \\ \cos b &= \cos \beta \sin a \\ \cos \gamma &= -\cot b \cot a \\ \cos \gamma &= -\cos \alpha \cos \beta \\ \cos a &= -\cot \gamma \tan \beta \\ \cos a &= \sin b \cos \alpha \\ \sin \beta &= \cot a \tan \alpha \\ \sin \beta &= \sin \gamma \sin b \end{aligned}$$