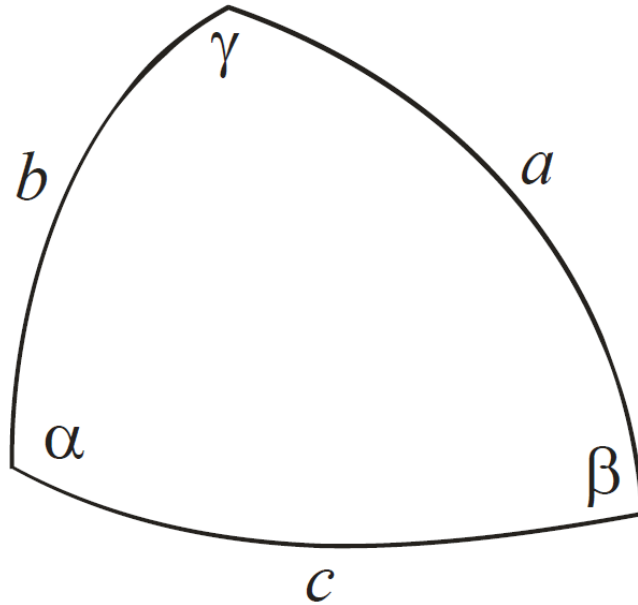


VAJA 9 – DEL 1: SFERNA TRIGONOMETRIJA – SPLOŠNI SFERNI TRIKOTNIK



Kosinusni izrek za stranice

$$\begin{aligned}\cos a &= \cos b \cos c + \sin b \sin c \cos \alpha \\ \cos b &= \cos a \cos c + \sin a \sin c \cos \beta \\ \cos c &= \cos a \cos b + \sin a \sin b \cos \gamma\end{aligned}$$

Kosinusni izrek za kote

$$\begin{aligned}\cos \alpha &= -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a \\ \cos \beta &= -\cos \alpha \cos \gamma + \sin \alpha \sin \gamma \cos b \\ \cos \gamma &= -\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c\end{aligned}$$

Sinusni izrek

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}$$

Kotangensni izrek

$$\begin{aligned}\cot a \sin b &= \cos b \cos \gamma + \cot \alpha \sin \gamma \\ \cot b \sin c &= \cos c \cos \alpha + \cot \beta \sin \alpha \\ \cot c \sin b &= \cos b \cos \alpha + \cot \gamma \sin \alpha \\ \cot a \sin c &= \cos c \cos \beta + \cot \alpha \sin \beta \\ \cot b \sin a &= \cos a \cos \gamma + \cot \beta \sin \gamma \\ \cot c \sin a &= \cos a \cos \beta + \cot \gamma \sin \beta\end{aligned}$$

Napierjeve enačbe (analogije)

$$\tan \frac{\alpha + \beta}{2} = \frac{\cos \frac{a - b}{2}}{\cos \frac{a + b}{2}} \cot \frac{\gamma}{2}$$

$$\tan \frac{\alpha - \beta}{2} = \frac{\sin \frac{a - b}{2}}{\sin \frac{a + b}{2}} \cot \frac{\gamma}{2}$$

$$\tan \frac{\alpha + \gamma}{2} = \frac{\cos \frac{a - c}{2}}{\cos \frac{a + c}{2}} \cot \frac{\beta}{2}$$

$$\tan \frac{\alpha - \gamma}{2} = \frac{\sin \frac{a - c}{2}}{\sin \frac{a + c}{2}} \cot \frac{\beta}{2}$$

$$\tan \frac{\beta + \gamma}{2} = \frac{\cos \frac{b - c}{2}}{\cos \frac{b + c}{2}} \cot \frac{\alpha}{2}$$

$$\tan \frac{\beta - \gamma}{2} = \frac{\sin \frac{b - c}{2}}{\sin \frac{b + c}{2}} \cot \frac{\alpha}{2}$$

$$\tan \frac{a + b}{2} = \frac{\cos \frac{\alpha - \beta}{2}}{\cos \frac{\alpha + \beta}{2}} \tan \frac{c}{2}$$

$$\tan \frac{a - b}{2} = \frac{\sin \frac{\alpha - \beta}{2}}{\sin \frac{\alpha + \beta}{2}} \tan \frac{c}{2}$$

$$\tan \frac{a + c}{2} = \frac{\cos \frac{\alpha - \gamma}{2}}{\cos \frac{\alpha + \gamma}{2}} \tan \frac{b}{2}$$

$$\tan \frac{a - c}{2} = \frac{\sin \frac{\alpha - \gamma}{2}}{\sin \frac{\alpha + \gamma}{2}} \tan \frac{b}{2}$$

$$\tan \frac{b + c}{2} = \frac{\cos \frac{\beta - \gamma}{2}}{\cos \frac{\beta + \gamma}{2}} \tan \frac{a}{2}$$

$$\tan \frac{b - c}{2} = \frac{\sin \frac{\beta - \gamma}{2}}{\sin \frac{\beta + \gamma}{2}} \tan \frac{a}{2}$$