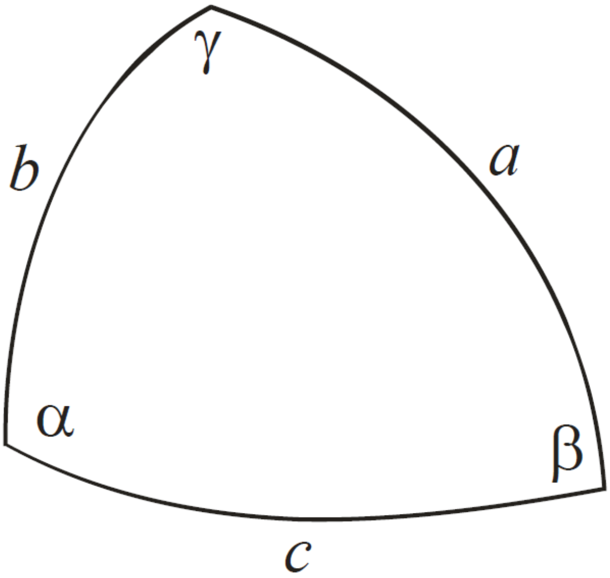
VAJA 9 – SFERNA TRIGONOMETRIJA

1 SPLOŠNI SFERNI TRIKOTNIK



Kosinusni izrek za stranice

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos \beta$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma$$

Kosinusni izrek za kote

$$\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a$$
$$\cos \beta = -\cos \alpha \cos \gamma + \sin \alpha \sin \gamma \cos b$$
$$\cos \gamma = -\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c$$

Sinusni izrek

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}$$

Kotangensni izrek

$$\begin{aligned}
\cot a \sin b &= \cos b \cos \gamma + \cot \alpha \sin \gamma \\
\cot b \sin c &= \cos c \cos \alpha + \cot \beta \sin \alpha \\
\cot c \sin b &= \cos b \cos \alpha + \cot \gamma \sin \alpha \\
\cot a \sin c &= \cos c \cos \beta + \cot \alpha \sin \beta \\
\cot b \sin a &= \cos a \cos \gamma + \cot \beta \sin \gamma \\
\cot c \sin a &= \cos a \cos \beta + \cot \gamma \sin \beta
\end{aligned}$$

Napierjeve enačbe (analogije)

$$\tan\frac{\alpha+\beta}{2} = \frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{\gamma}{2} \qquad \tan\frac{\alpha-\beta}{2} = \frac{\sin\frac{a-b}{2}}{\sin\frac{a+b}{2}}\cot\frac{\gamma}{2}$$

$$\tan\frac{\alpha+\gamma}{2} = \frac{\cos\frac{a-c}{2}}{\cos\frac{a+c}{2}}\cot\frac{\beta}{2} \qquad \tan\frac{\alpha-\gamma}{2} = \frac{\sin\frac{a-c}{2}}{\sin\frac{a+c}{2}}\cot\frac{\beta}{2}$$

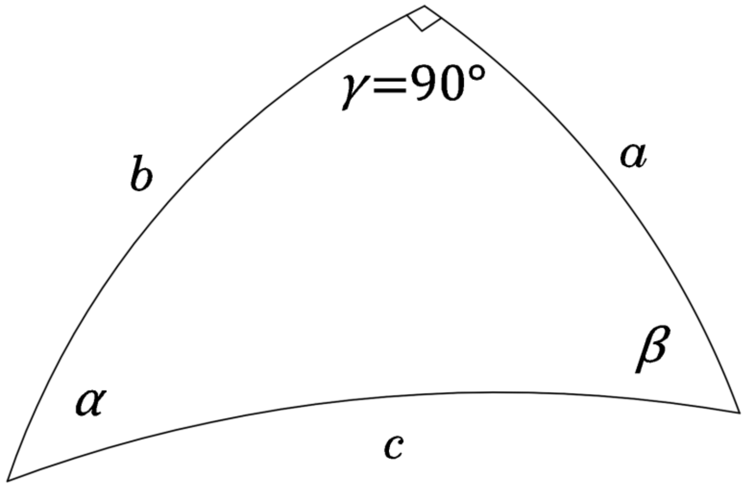
$$\tan\frac{\beta+\gamma}{2} = \frac{\cos\frac{b-c}{2}}{\cos\frac{b+c}{2}}\cot\frac{\alpha}{2} \qquad \tan\frac{\beta-\gamma}{2} = \frac{\sin\frac{b-c}{2}}{\sin\frac{b+c}{2}}\cot\frac{\alpha}{2}$$

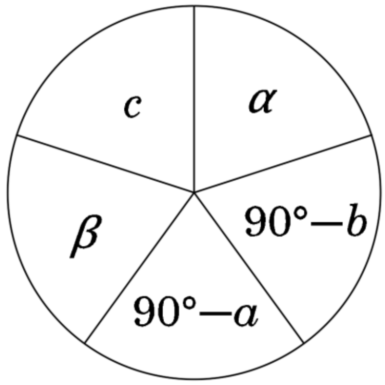
$$\tan\frac{a+b}{2} = \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}\tan\frac{c}{2} \qquad \tan\frac{a-b}{2} = \frac{\sin\frac{\alpha-\beta}{2}}{\sin\frac{\alpha+\beta}{2}}\tan\frac{c}{2}$$

$$\tan\frac{a+c}{2} = \frac{\cos\frac{\alpha-\gamma}{2}}{\cos\frac{\alpha+\gamma}{2}}\tan\frac{b}{2} \qquad \tan\frac{a-c}{2} = \frac{\sin\frac{\alpha-\gamma}{2}}{\sin\frac{\alpha+\gamma}{2}}\tan\frac{b}{2}$$

$$\tan\frac{b+c}{2} = \frac{\cos\frac{\beta-\gamma}{2}}{\cos\frac{\beta+\gamma}{2}}\tan\frac{a}{2} \qquad \tan\frac{b-c}{2} = \frac{\sin\frac{\beta-\gamma}{2}}{\sin\frac{\beta+\gamma}{2}}\tan\frac{a}{2}$$

2 PRAVOKOTNI SFERNI TRIKOTNIK





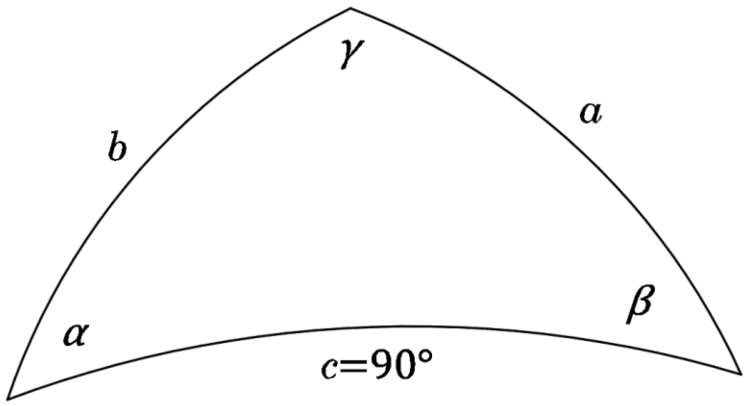
Napierjevo pravilo

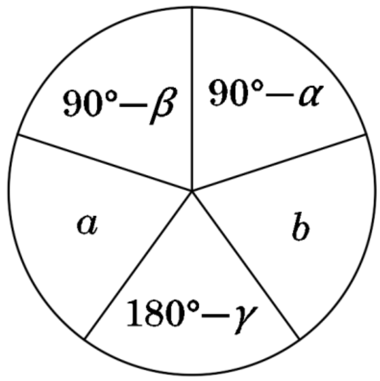
Kosinus izbranega elementa v "krogu" (shema desno) je enak (shema zgoraj desno):

i) produktu kotangensov sosednjih elementov,
ii) produktu sinusov nasprotnih elementov.

Osnovna oblika izraza	**Poenostavljena oblika izraza**
$\cos c = \cot\beta \cot\alpha$	$\cos c = \cot\beta \cot\alpha$
$\cos c = \sin(90° - a) \sin(90° - b)$	$\cos c = \cos a \cos b$
$\cos\alpha = \cot c \cot(90° - b)$	$\cos\alpha = \cot c \tan b$
$\cos\alpha = \sin\beta \sin(90° - \alpha)$	$\cos\alpha = \sin\beta \cos\alpha$
$\cos(90° - b) = \cot\alpha \cot(90° - a)$	$\sin b = \cot\alpha \tan a$
$\cos(90° - b) = \sin c \sin\beta$	$\sin b = \sin c \sin\beta$
$\cos(90° - a) = \cot(90° - b) \cot\beta$	$\sin a = \tan b \cot\beta$
$\cos(90° - a) = \sin\alpha \sin c$	$\sin a = \sin\alpha \sin c$
$\cos\beta = \cot(90° - a) \cot c$	$\cos\beta = \tan a \cot c$
$\cos\beta = \sin(90° - b) \sin\alpha$	$\cos\beta = \cos b \sin\alpha$

3 PRAVOSTRANIČNI SFERNI TRIKOTNIK





Napierjevo pravilo

Kosinus izbranega elementa v "krogu" (shema desno) je enak (shema zgoraj desno):

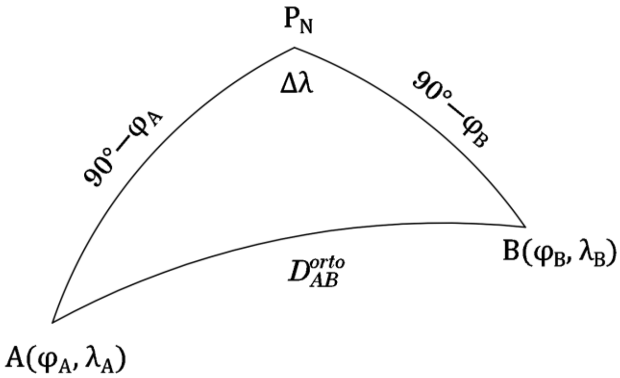
i) produktu kotangensov sosednjih elementov,
ii) produktu sinusov nasprotnih elementov.

Osnovna oblika izraza	**Poenostavljena oblika izraza**
$\cos(90° - \alpha) = \cot(90° - \beta)\cot b$	$\sin\alpha = \tan\beta\cot b$
$\cos(90° - \alpha) = \sin a\sin(180° - \gamma)$	$\sin\alpha = \sin a\sin\gamma$
$\cos b = \cot(90° - \alpha)\cot(180° - \gamma)$	$\cos b = -\tan\alpha\cot\gamma$
$\cos b = \sin(90° - \beta)\sin a$	$\cos b = \cos\beta\sin a$
$\cos(180° - \gamma) = \cot b\cot a$	$\cos\gamma = -\cot b\cot a$
$\cos(180° - \gamma) = \sin(90° - \alpha)\sin(90° - \beta)$	$\cos\gamma = -\cos\alpha\cos\beta$
$\cos a = \cot(180° - \gamma)\cot(90° - \beta)$	$\cos a = -\cot\gamma\tan\beta$
$\cos a = \sin b\sin(90° - \alpha)$	$\cos a = \sin b\cos\alpha$
$\cos(90° - \beta) = \cot a\cot(90° - \alpha)$	$\sin\beta = \cot a\tan\alpha$
$\cos(90° - \beta) = \sin(180° - \gamma)\sin b$	$\sin\beta = \sin\gamma\sin b$

4 ORTODROMA

Ortodroma oziroma geodetska linija je najkrajša razdalja med dvema točkama na krogli. Ortodroma je krajši lok velikega kroga skozi dani točki na površju krogle. Kot pot plovbe/leta ni primerna saj bi moralo plovilo/letalo za pot po ortodromi stalno spreminjati smer (kurz) poti.

<u>dano</u>: $A(\varphi_A, \lambda_A), B(\varphi_B, \lambda_B)$
<u>iščemo</u>: D_{AB}^{orto}



Dolžino ortodrome v kotnih enotah izračunamo z uporabo kosinusnega izrek za stranice:

$$\cos D_{AB}^{orto} = \cos(90° - \varphi_A)\cos(90° - \varphi_B) + \sin(90° - \varphi_A)\sin(90° - \varphi_B)\cos\Delta\lambda$$

kjer je:

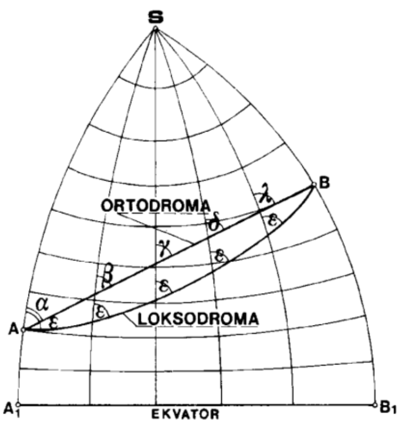
$$\Delta\lambda = |\lambda_B - \lambda_A|$$

Poldnevniki so veliki krogi → loki poldnevnikov so torej ortodrome. Vzporedniki so mali krogi (razen ekvator) → loki vzporednikov NISO ortodrome (razen loki ekvatorja).

5 LOKSODROMA

Loksodroma je pot med dvema točkama, ki vse meridiane seka pod enakim kotom. Plovilu/letalu, ki potuje po loksodromi, ni potrebno spreminjati kurza poti.

<u>dano</u>: začetna točka $A(\varphi_A, \lambda_A)$, končna točka $B(\varphi_B, \lambda_B)$
<u>iščemo</u>: D_{AB}^{lokso}, α



Loksodroma ni lok velikega kroga, zato ne moremo uporabiti enačb sferne trigonometrije. Azimut loksodrome (smer poti oziroma kurz) α (na zgornji skici označen z ε) izračunamo kot:

$$\cot\alpha = \frac{1}{(\lambda_B - \lambda_A)\frac{\pi}{180°}} \ln\left(\frac{\tan\left(45° + \frac{\varphi_B}{2}\right)}{\tan\left(45° + \frac{\varphi_A}{2}\right)}\right)$$

Za končni izračun azimuta je potrebno določiti še njegov kvadrant:

$\boldsymbol{\lambda_A - \lambda_B}$	**cot** $\boldsymbol{\alpha}$	**kvadrant**	$\boldsymbol{\alpha}$
$-$	$+$	I. kvadrant	α
$-$	$-$	II. kvadrant	$\alpha + 180°$
$+$	$+$	III. kvadrant	$\alpha + 180°$
$+$	$-$	IV. kvadrant	$\alpha + 360°$

Dolžino loksodrome v dolžinskih enotah izračunamo kot:

$$D_{AB}^{loks} = R\frac{(\varphi_B - \varphi_A)}{\cos\alpha}\frac{\pi}{180°}$$