Requirements for Generating

A Geometrically Correct Point Cloud

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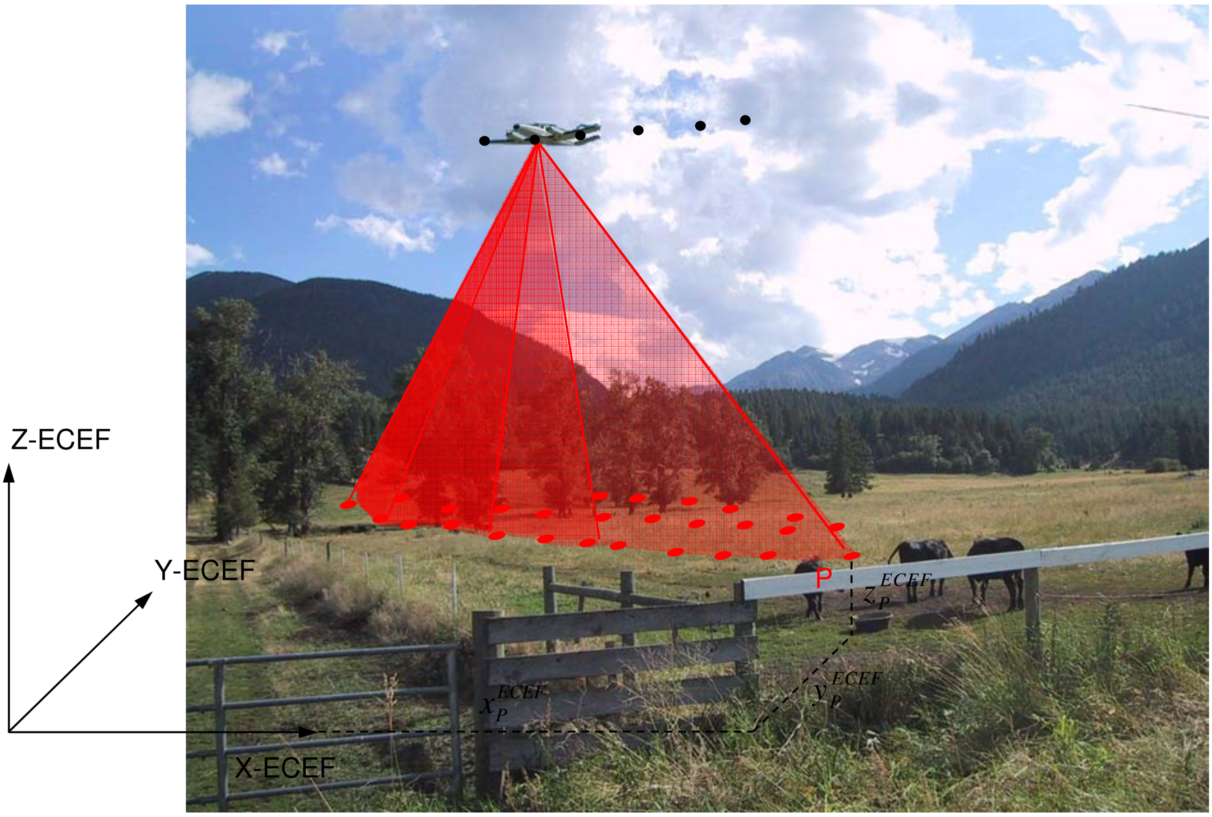
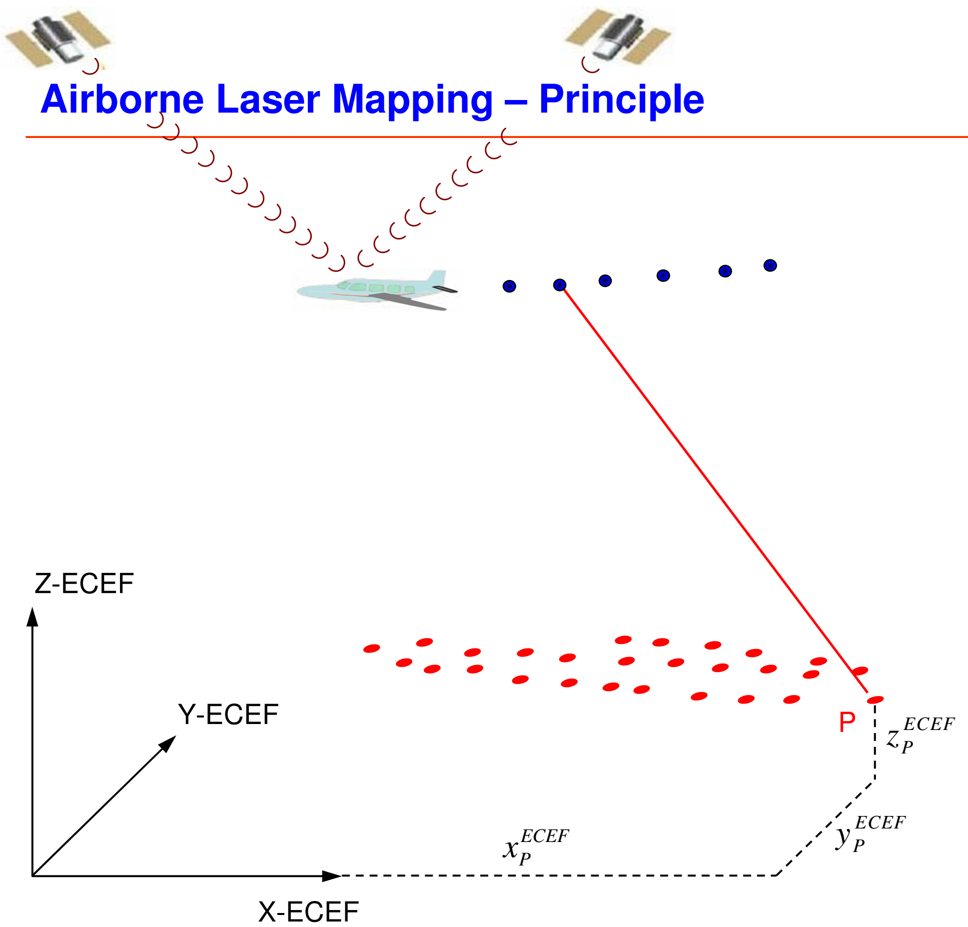
AGENDA

1. **Introduction**
   - Principle of airborne laser mapping (ALM)
   - Characteristics of ALM
   - Products and product quality
2. **Mathematical model for laser point computation**
   - Functional model (basic)
   - Stochastic model
   - Lab calibration
   - Effects of model (calibration) parameters on laser points
   - Extended functional model
3. **Laser point block adjustment**
   - Description/definition
   - Mathematical model
   - Parameter determinability
   - Empirical results
4. **Summary**

Airborne Laser Mapping – Principle



Airborne Laser Mapping – Principle



Sensor Orientation

- Position by GPS
- Attitudes by INS

Sensor Measurements

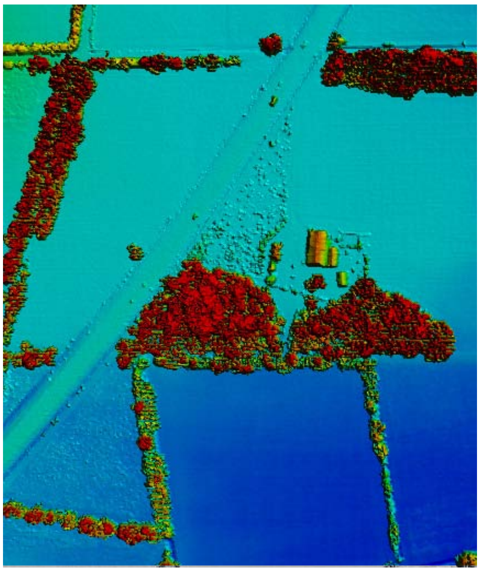
- Laser ranges
- Scanner angles

Airborne Laser Mapping – Characteristics

- Distance measurements to almost any type of surface
- Ability to measure several elevations along same direction (multiple returns)
- Detected signal strength of the target-reflected laser-emitted pulse (intensity)

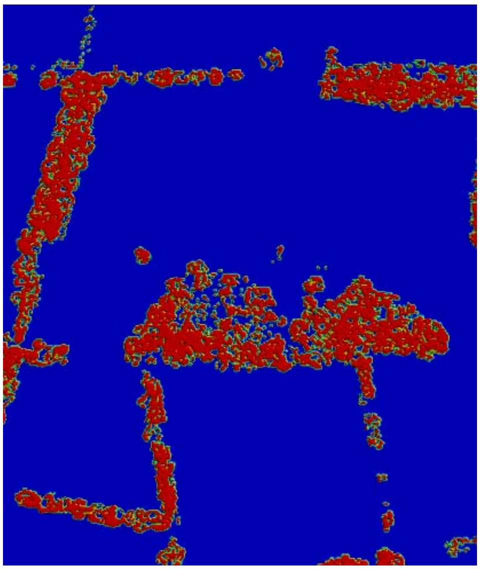
Laser Point Information

Height



Intensity

Return



Airborne Laser Mapping – Instrument Development

1993

- 2 kHz
- 1000 m max. AGL height
- ± 20 degree scan angle
- first or last return
- 4 – 5 m point spacing
  (single overpass @ 1000 m AGL)

2007

- up to ~170 kHz
- up to 4500 m max. AGL height
- up to ± 32 degree scan angle
- multiple returns to full wave form
- 8 – 16 bit intensity values
- < 1 m point spacing
  (single overpass @ 1000 m AGL)

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Airborne Laser Mapping – Products

Service

ALM survey flight

Key products

Laser point cloud

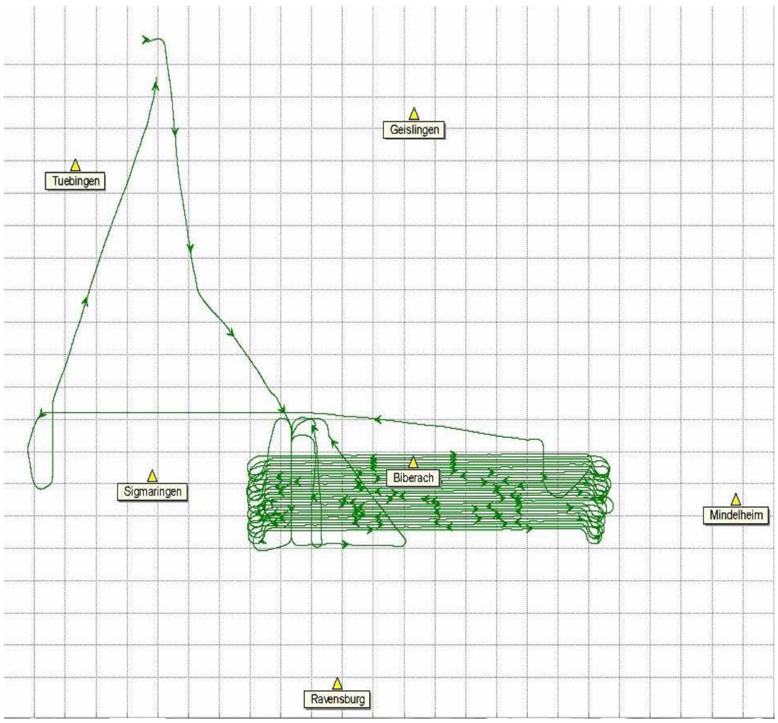
Classified laser point cloud

Derivative products

E.g. digital elevation models

Intensity images

Digital photos, ortho-photos



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Airborne Laser Mapping – Product Quality

Service

ALM survey flight

Key products

Laser point cloud

Classified laser point cloud

Derivative products

E.g. digital elevation models

Intensity images

Digital photos, ortho-photos

General quality attributes

Schedule, completeness, point density, etc.

Accuracy

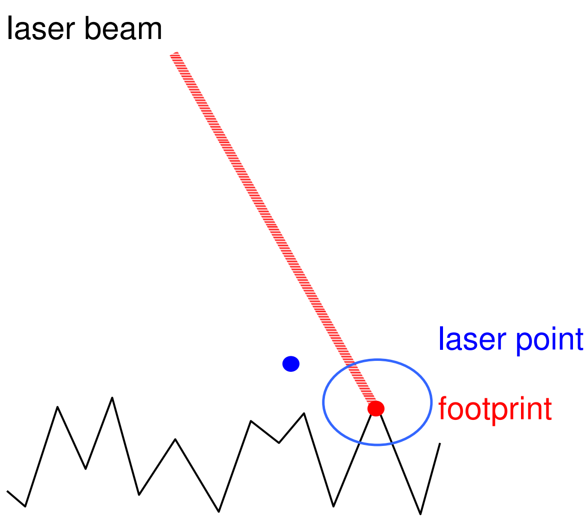
Laser point accuracy

Classification accuracy

DTM accuracy

Laser Point Accuracy

- Closeness of the computed laser point position to the position of the true laser footprint
- Expressed as standard deviations of the laser point coordinates



Classification Accuracy

- The degree of correctness of assigned attributes
- Expressed in probability for the correctness of the assigned attribute

Geometry Attributes

Plane, line, ground, etc.

Object Attributes

Building, street, tree, etc.

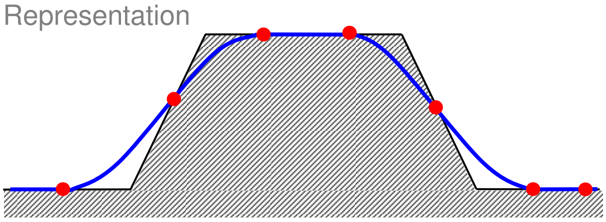
Material Attributes

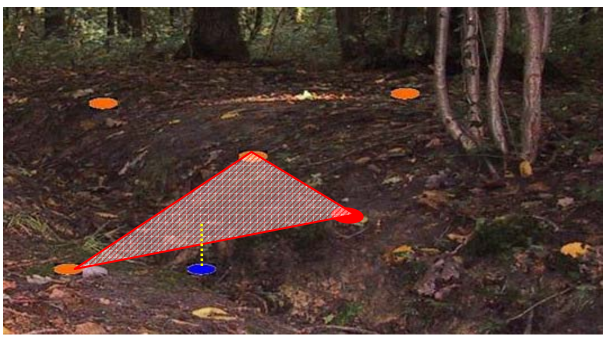
Asphalt, concrete, water, etc.

DTM Accuracy

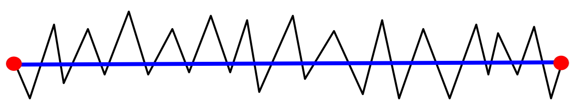
- Vertical closeness of the DTM surface to the true physical terrain surface
- Expressed as RMS of the differences between the interpolated height and the measured height of check points

Representation





Surface Roughness



ALM Product Accuracies

Key products

- Laser point cloud
- Classified laser point cloud

Derivative products

- E.g. digital elevation models

} have quite distinct accuracy characteristics

Currently, ALM products are usually only empirically evaluated for accuracy by comparison to ground truth data.

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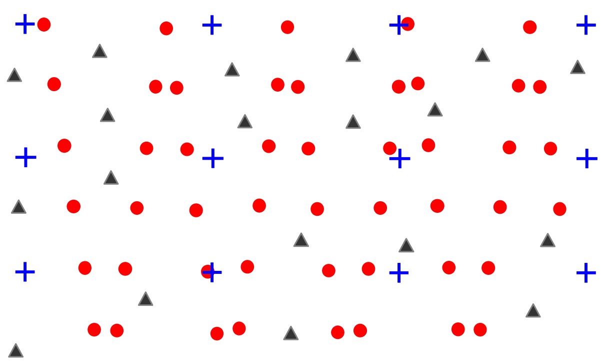
Empirical Accuracy Tests



True laser footprint position is NOT known !

Therefore:

No direct point to point correspondence between laser points (●) and control points (▲).

Thus:

Correspondence via surface, e.g. DEM (+).

$$\Delta H = H_{LP} - H_{CP\text{-}DEM}$$

minimum ΔH

maximum ΔH

mean ΔH

rms ΔH

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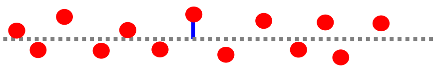
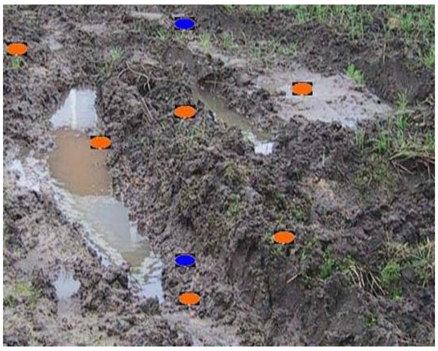
Empirical Accuracy Interpretation

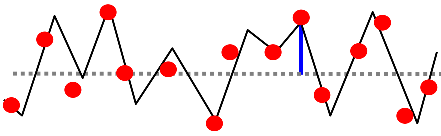
Asphalt



rms Δh = 0.05 m

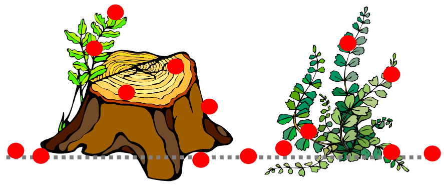
Farmland, meadows, …





rms Δh = 0.23 m

Forest



rms Δh = 0.41 m

Mathematical Model

Observations ⇒ **Mathematical model** ⇒ **Parameters**

$$\mathbf{l}^T = [r\ \Theta\ x\ y\ z\ \eta\ \chi\ \alpha]$$

$$\mathbf{C}_{ll} = \begin{bmatrix} \sigma_r^2 & & \\ & \sigma_\Theta^2 & \\ & & \ddots \end{bmatrix}$$

⇒

Functional model

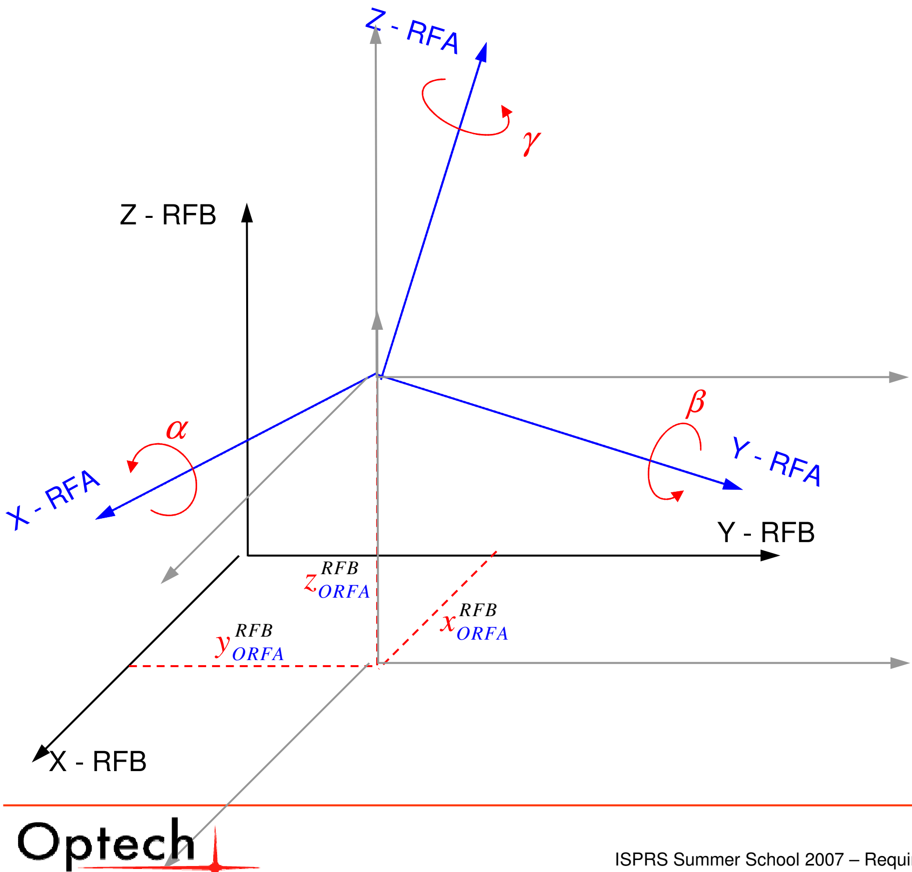
Stochastic model

⇒

$$\mathbf{X}_{LP}^T = [x_{LP}\quad y_{LP}\quad z_{LP}]$$

$$\mathbf{C}_{XX} = \begin{bmatrix} \sigma_x^2 & c_{xy} & c_{xz} \\ c_{xy} & \sigma_y^2 & c_{yz} \\ c_{xz} & c_{yz} & \sigma_z^2 \end{bmatrix}$$

[Geo] Referencing – Principle



Orientation Parameters, or

Orientation Elements, or

Transformation Parameters, or …

A set of values describing the relationship between two reference frames.

- 3 rotation angles
- 3 translations

RFA = <u>R</u>eference <u>F</u>rame <u>A</u>

RFB = <u>R</u>eference <u>F</u>rame <u>B</u>

Rotation Matrices

$$\mathbf{R}_X(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \qquad \mathbf{R}_Y(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad \mathbf{R}_Z(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The rotations are positive in the counterclockwise sense as viewed along the axis toward the origin (right-hand-rule).

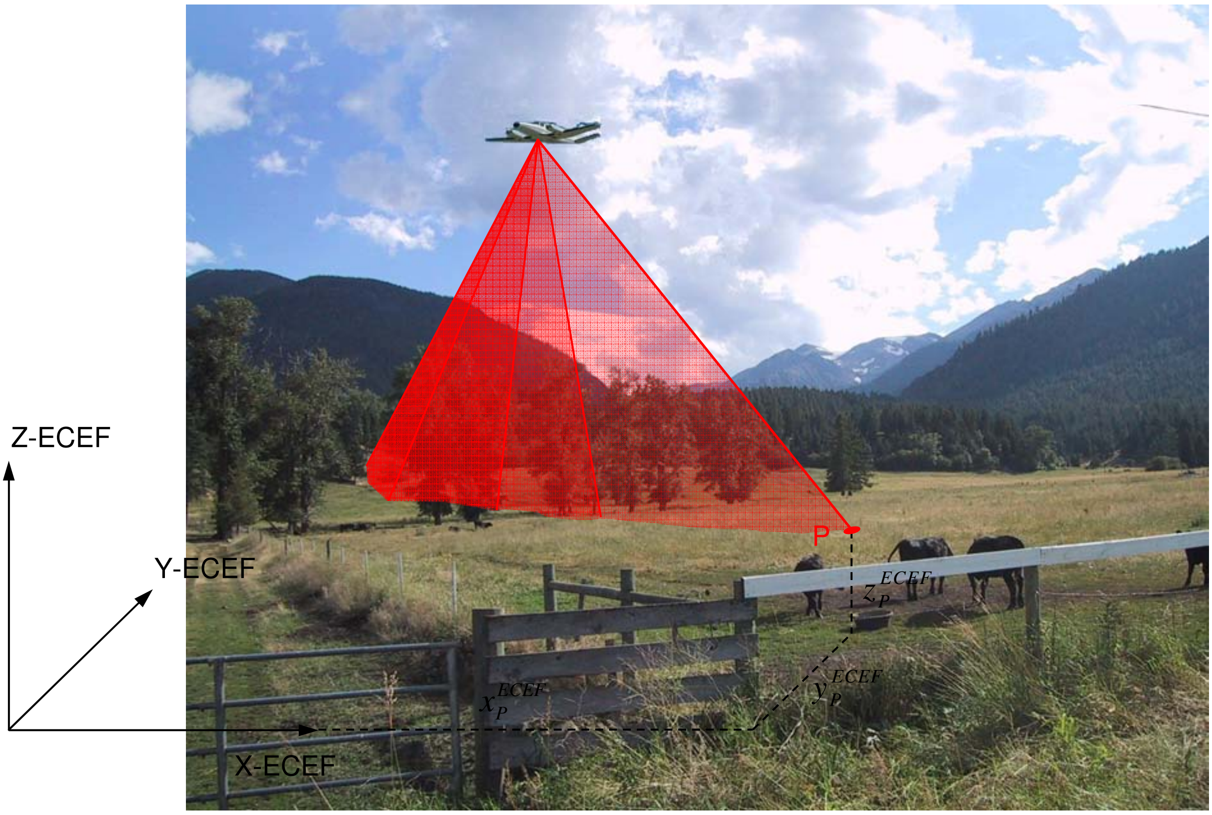
The rotation matrices are orthogonal:

$$\mathbf{R}^{-1}(\Theta) = \mathbf{R}^{T}(\Theta) = \mathbf{R}(-\Theta)$$

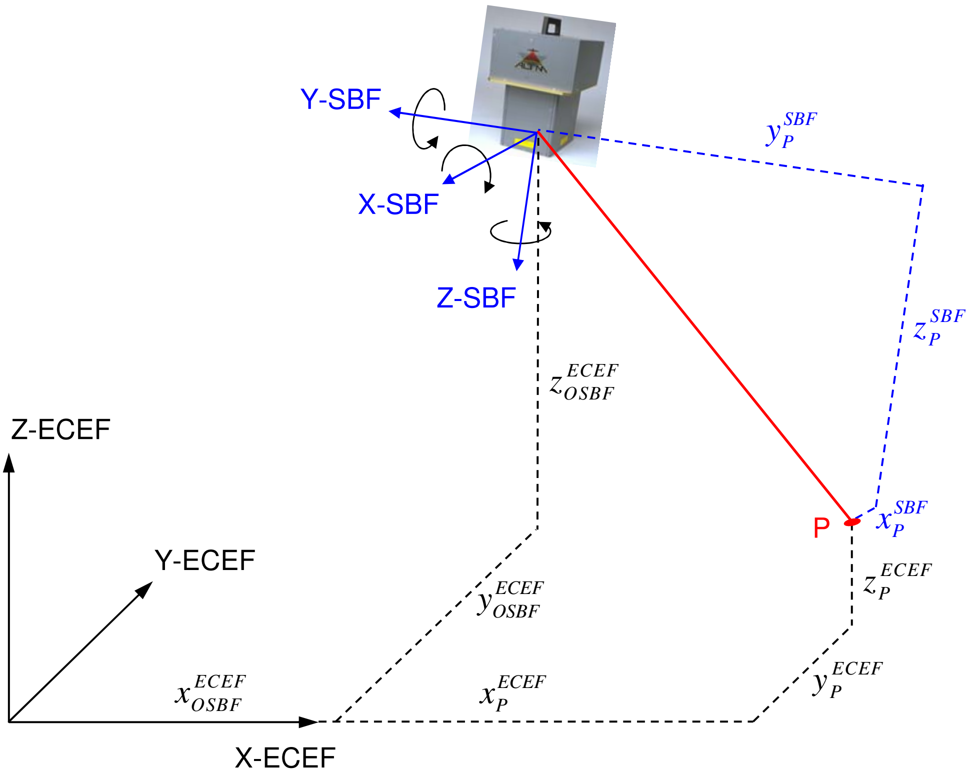
Thus:

$$(\mathbf{R}_Z(\gamma)\mathbf{R}_Y(\beta)\mathbf{R}_X(\alpha))^{-1} = (\mathbf{R}_Z(\gamma)\mathbf{R}_Y(\beta)\mathbf{R}_X(\alpha))^{T} = \mathbf{R}_X(-\alpha)\mathbf{R}_Y(-\beta)\mathbf{R}_Z(-\gamma)$$

Laser Point Computation – Functional Model



Laser Point Computation – Functional Model



$$\mathbf{X}_P^{ECEF} = \mathbf{X}_{OSBF}^{ECEF} + \mathbf{R}_{SBF}^{ECEF} \cdot \mathbf{X}_P^{SBF}$$

with:

$$\mathbf{X}_P^{ECEF} = \left[x_P^{ECEF} \quad y_P^{ECEF} \quad z_P^{ECEF} \right]^T$$

$$\mathbf{X}_P^{SBF} = \left[x_P^{SBF} \quad y_P^{SBF} \quad z_P^{SBF} \right]^T$$

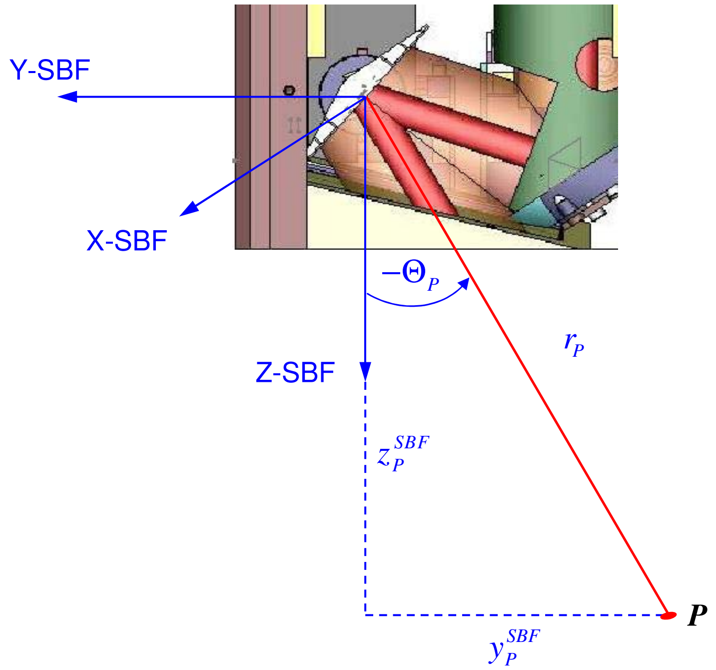
$$\mathbf{X}_{OSBF}^{ECEF} = \left[x_{OSBF}^{ECEF} \quad y_{OSBF}^{ECEF} \quad z_{OSBF}^{ECEF} \right]^T$$

$$\mathbf{R}_{SBF}^{ECEF} = \mathbf{R}_{SBF}^{ECEF}(\alpha, \beta, \gamma)$$

SBF = Sensor Body Frame

ECEF = Earth Centered…

Sensor Model



SBF = Sensor Body Frame

$$\mathbf{X}_P^{SBF} = \begin{bmatrix} 0 \\ r_p \sin\Theta_P \\ r_p \cos\Theta_P \end{bmatrix} = \mathbf{R}_X(\Theta_P) \cdot \begin{bmatrix} 0 \\ 0 \\ r_p \end{bmatrix}$$

with:

$$\mathbf{X}_P^{SBF} = \begin{bmatrix} x_P^{SBF} & y_P^{SBF} & z_P^{SBF} \end{bmatrix}^T$$

Θ_P = scan angle

r_P = range

Observation model:

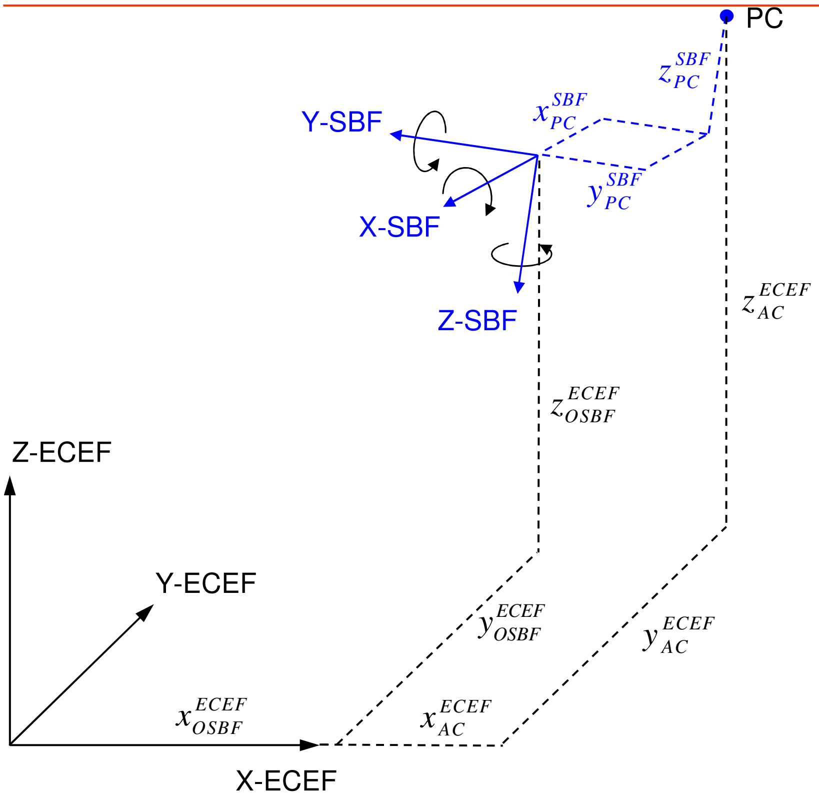
$\Theta_P = s_\Theta \cdot \Theta_{obs} + \Delta\Theta$

$r_P = r_{obs} + \Delta r$

$s_\Theta, \Delta\Theta$ = scan angle corrections

Δr = range corrections

Geo-Referencing - Position



$$\mathbf{X}_P^{ECEF} = \mathbf{X}_{OSBF}^{ECEF} + \mathbf{R}_{SBF}^{ECEF} \cdot \mathbf{X}_P^{SBF}$$

with:

$\mathbf{X}_P^{ECEF} = \begin{bmatrix} x_P^{ECEF} & y_P^{ECEF} & z_P^{ECEF} \end{bmatrix}^T$

$\mathbf{X}_P^{SBF} = \surd$

$\mathbf{X}_{OSBF}^{ECEF} = \mathbf{X}_{PC}^{ECEF} - \mathbf{R}_{SBF}^{ECEF} \cdot \mathbf{X}_{PC}^{SBF}$

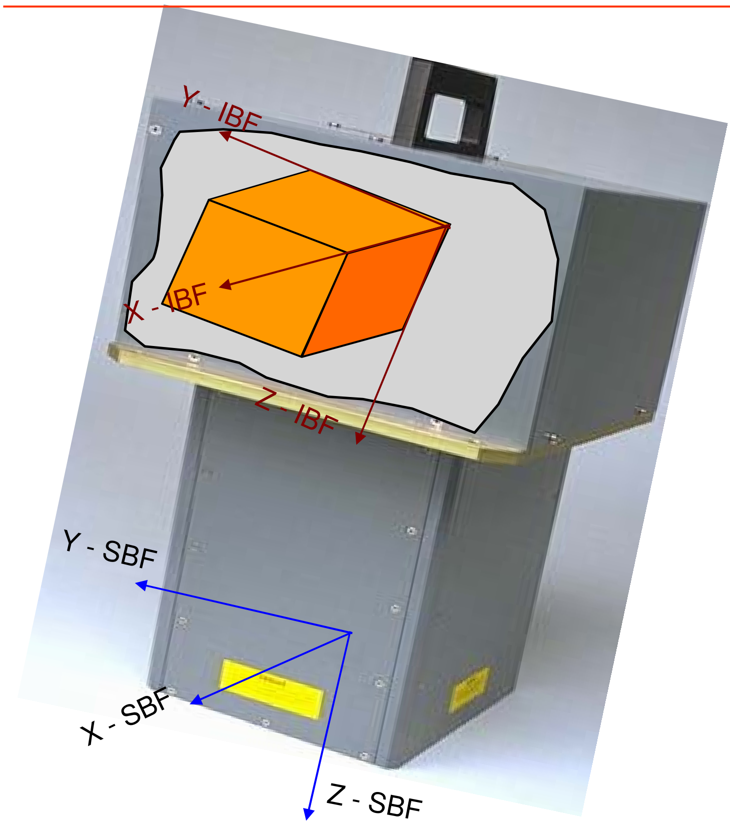
$\mathbf{R}_{SBF}^{ECEF} =$ **?**

PC = Positioning Center

SBF = Sensor Body Frame

ECEF = Earth Centered…

Geo-Referencing - Orientation

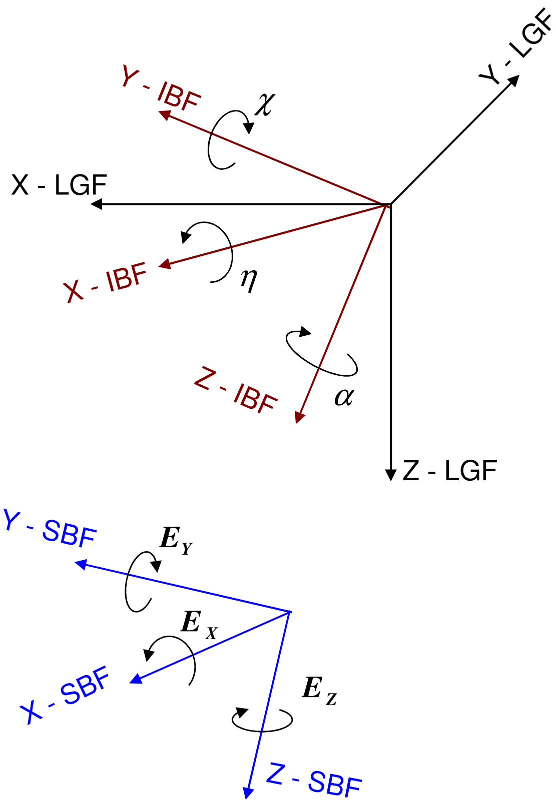


$\mathbf{R}_{SBF}^{ECEF} = $ **?**

IBF = IMU Body Frame

SBF = Sensor Body Frame

Geo-Referencing – Orientation



$$\mathbf{R}_{SBF}^{ECEF} = \mathbf{R}_{LGF}^{ECEF} \cdot \mathbf{R}_{IBF}^{LGF} \cdot \mathbf{R}_{SBF}^{IBF}$$

with:

$$\mathbf{R}_{SBF}^{IBF} = \mathbf{R}_{Z}^{T}(E_Z) \cdot \mathbf{R}_{Y}^{T}(E_Y) \cdot \mathbf{R}_{X}^{T}(E_X)$$

$$\mathbf{R}_{IBF}^{LGF} = \mathbf{R}_{Z}^{T}(\alpha) \cdot \mathbf{R}_{Y}^{T}(\chi) \cdot \mathbf{R}_{X}^{T}(\eta)$$

$$\mathbf{R}_{LGF}^{ECEF} = \mathbf{R}_{Z}(-\lambda) \cdot \mathbf{R}_{Y}(\varphi + \tfrac{\pi}{2})$$

E_X, E_Y, E_Z = SBF-IBF relative orientation

η, χ, α = roll, pitch, heading

φ, λ = latitude, longitude

LGF = Local Geodetic Frame

IBF = IMU Body Frame

SBF = Sensor Body Frame

Laser Point Computation – Functional Model (Basic)

$$\mathbf{X}_{P}^{ECEF} = \mathbf{X}_{PC}^{ECEF} + \mathbf{R}_{LGF}^{ECEF} \cdot \mathbf{R}_{IBF}^{LGF} \cdot \mathbf{R}_{SBF}^{IBF} \cdot \left(\mathbf{X}_{P}^{SBF} - \mathbf{X}_{PC}^{SBF} \right)$$

with:

$$\mathbf{R}_{SBF}^{IBF} = \mathbf{R}_{Z}^{T}(E_Z) \cdot \mathbf{R}_{Y}^{T}(E_Y) \cdot \mathbf{R}_{X}^{T}(E_X)$$

$$\mathbf{R}_{IBF}^{LGF} = \mathbf{R}_{Z}^{T}(\alpha) \cdot \mathbf{R}_{Y}^{T}(\chi) \cdot \mathbf{R}_{X}^{T}(\eta)$$

$$\mathbf{R}_{LGF}^{ECEF} = \mathbf{R}_{Z}(-\lambda) \cdot \mathbf{R}_{Y}(\varphi + \tfrac{\pi}{2})$$

E_X, E_Y, E_Z = SBF-IBF relative orientation

η, χ, α = Roll, pitch, heading

φ, λ = Latitude, longitude

$\mathbf{X}_{PC}^{ECEF}$ = Sensor position center

$\mathbf{X}_{PC}^{SBF}$ = Position center eccentricities

$\mathbf{X}_{P}^{SBF}$ = Laser point in sensor body frame

e.g.

Sensor Model - ALS

$$\mathbf{X}_{P}^{SBF} = \begin{bmatrix} 0 \\ r_p \sin\Theta_P \\ r_p \cos\Theta_P \end{bmatrix} = \mathbf{R}_X(\Theta_P) \cdot \begin{bmatrix} 0 \\ 0 \\ r_p \end{bmatrix}$$

with:

Θ_P = scan angle

r_P = range

SBF = Sensor Body Frame

IBF = IMU Body Frame

LGF = Local Geodetic Frame

ECEF = Earth Centered Earth Fixed

Laser Point Computation – Functional Model (Basic)

Observations

GPS antenna center position $\mathbf{X}_{PC}^{ECEF}$

IMU roll, pitch, heading $\eta, \chi, \alpha \Rightarrow \mathbf{R}_{IBF}^{LGF}$

ALS scan angle, laser range $\Theta, r \Rightarrow \mathbf{X}_{P}^{SBF}$

Mounting parameters

ALS-IMU relative orientation $E_X, E_Y, E_Z \Rightarrow \mathbf{R}_{SBF}^{IBF}$

ALS-Position center eccentricities $\mathbf{X}_{PC}^{SBF}$

Corrections

Observation corrections $\Delta r, \Delta\Theta, \Delta s$

Laser Point Computation – Stochastic Model

$\mathbf{C}_{XX}^{ECEF} = \mathbf{?}$

for

$$\mathbf{X}_{P}^{ECEF} = \mathbf{X}_{PC}^{ECEF} + \mathbf{R}_{LGF}^{ECEF} \cdot \mathbf{R}_{IBF}^{LGF} \cdot \mathbf{R}_{SBF}^{IBF} \cdot \left(\mathbf{X}_{P}^{SBF} - \mathbf{X}_{PC}^{SBF} \right)$$

- $\mathbf{X}_{P}^{SBF}$ → Indirect observations $\Rightarrow \mathbf{C}_{XX}^{SBF}$
- $\mathbf{R}_{IBF}^{LGF}$ → Indirect observations $\Rightarrow \mathbf{C}_{IMU}^{ECEF}$
- $\mathbf{X}_{PC}^{ECEF}$ → Indirect observations $\Rightarrow \mathbf{C}_{GPS}^{ECEF}$

$\mathbf{C}_{GPS}^{ECEF}$ = Covariance matrix of antenna position

$\mathbf{C}_{IMU}^{ECEF}$ = Covariance matrix of roll, pitch, heading

$\mathbf{C}_{XX}^{SBF}$ = Covariance matrix of laser point in SBF

Covariance Law

Functional Model

$$\mathbf{x} = \mathbf{f}(\mathbf{l})$$

with:

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_u \end{bmatrix}^T$$

$$\mathbf{l} = \begin{bmatrix} l_1 & l_2 & \cdots & l_n \end{bmatrix}^T$$

and

$$x_1 = f_1(l_1 \quad l_2 \quad \cdots \quad l_n)$$

$$x_2 = f_2(l_1 \quad l_2 \quad \cdots \quad l_n)$$

$$\vdots$$

$$x_u = f_u(l_1 \quad l_2 \quad \cdots \quad l_n)$$

Stochastic Model

$$\mathbf{C}_{XX} = \mathbf{J} \cdot \mathbf{C}_{ll} \cdot \mathbf{J}^T$$

with:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x_1}{\partial l_1} & \frac{\partial x_1}{\partial l_2} & \cdots & \frac{\partial x_1}{\partial l_n} \\ \frac{\partial x_2}{\partial l_1} & \frac{\partial x_2}{\partial l_2} & \cdots & \frac{\partial x_2}{\partial l_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_u}{\partial l_1} & \frac{\partial x_u}{\partial l_2} & \cdots & \frac{\partial x_u}{\partial l_n} \end{bmatrix}$$ = Jacobian matrix

$\mathbf{C}_{XX}$ = Covariance matrix of parameters **x**

$\mathbf{C}_{ll}$ = Covariance matrix of observations **l**

Laser Point Computation – Stochastic Model

$$\mathbf{C}_{XX}^{ECEF} = \mathbf{C}_{GPS}^{ECEF} + \mathbf{J}_{IMU} \cdot \mathbf{C}_{IMU}^{IBF} \cdot \mathbf{J}_{IMU}^{T} + \mathbf{J}_{XX} \cdot \mathbf{C}_{XX}^{SBF} \cdot \mathbf{J}_{XX}^{T}$$

with:

$$\mathbf{J}_{IMU} = \begin{bmatrix} \frac{\delta \mathbf{X}_P^{ECEF}}{\delta \eta} & \frac{\delta \mathbf{X}_P^{ECEF}}{\delta \chi} & \frac{\delta \mathbf{X}_P^{ECEF}}{\delta \alpha} \end{bmatrix}$$

$$\mathbf{J}_{XX} = \frac{\delta \mathbf{X}_P^{ECEF}}{\delta \mathbf{X}_P^{SBF}}$$

$\mathbf{C}_{GPS}^{ECEF}$ = Covariance matrix of antenna position

$\mathbf{C}_{IMU}^{ECEF}$ = Covariance matrix of roll, pitch, heading

$\mathbf{C}_{XX}^{SBF}$ = Covariance matrix of laser point in SBF

$\mathbf{X}_P^{ECEF}$ = Laser point in ECEF

$\mathbf{X}_P^{SBF}$ = Laser point in SBF

Stochastic Sensor Model

$$\mathbf{C}_{XX}^{SBF} = \mathbf{J}_{ll} \cdot \mathbf{C}_{ll}^{SBF} \cdot \mathbf{J}_{ll}^{T}$$

with:

$$\mathbf{J}_{ll} = \begin{bmatrix} \frac{\delta \mathbf{X}_P^{SBF}}{\delta r_{obs}} & \frac{\delta \mathbf{X}_P^{SBF}}{\delta \Theta_{Obs}} \end{bmatrix}$$

$$\mathbf{C}_{ll}^{SBF} = \begin{bmatrix} \sigma_r^2 & \\ & \sigma_\Theta^2 \end{bmatrix}$$

σ_r = Standard dev. of laser range

σ_Θ = Standard dev. of scan angle

Stochastic Model – Error Propagation

Random Observation Errors

Laser range	σ_R = 0.050 m		
Scan angle (mechanical)	σ_Θ = 0.0025 deg		
GPS antenna position	σ_X = 0.050 m	σ_Y = 0.050 m	σ_Z = 0.080 m
IMU attitudes	σ_R = 0.005 deg	σ_P = 0.005 deg	σ_H = 0.020 deg

Resulting Random Errors for Laser Points (AGL 1000 m)

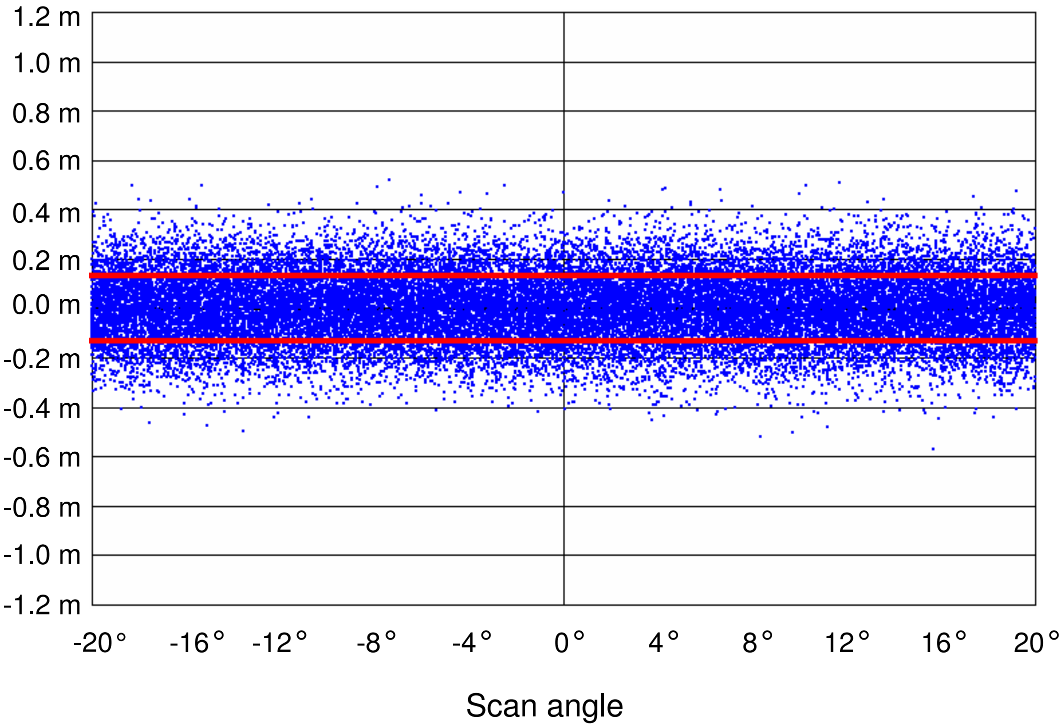
East (Cross-Flight)	σ_{min} = 0.133 m	σ_{mean} = 0.133 m	σ_{max} = 0.134 m
North (In-Flight)	σ_{min} = 0.101 m	σ_{mean} = 0.124 m	σ_{max} = 0.162 m
Height	σ_{min} = 0.094 m	σ_{mean} = 0.097 m	σ_{max} = 0.103 m

Resulting Random Errors for Laser Points (AGL 2000 m)

East (Cross-Flight)	σ_{min} = 0.252 m	σ_{mean} = 0.252 m	σ_{max} = 0.252 m
North (In-Flight)	σ_{min} = 0.182 m	σ_{mean} = 0.229 m	σ_{max} = 0.312 m
Height	σ_{min} = 0.094 m	σ_{mean} = 0.107 m	σ_{max} = 0.129 m

Stochastic Model – Example AGL 1000

Errors in Easting (Cross-Flight)



Empirical

max. ε = 0.518 m

min. ε = -0.569 m

mean ε = 0.000 m

rms ε = 0.133 m

$-\sigma \leq \varepsilon \leq \sigma$ = 68.2 %

Theoretical

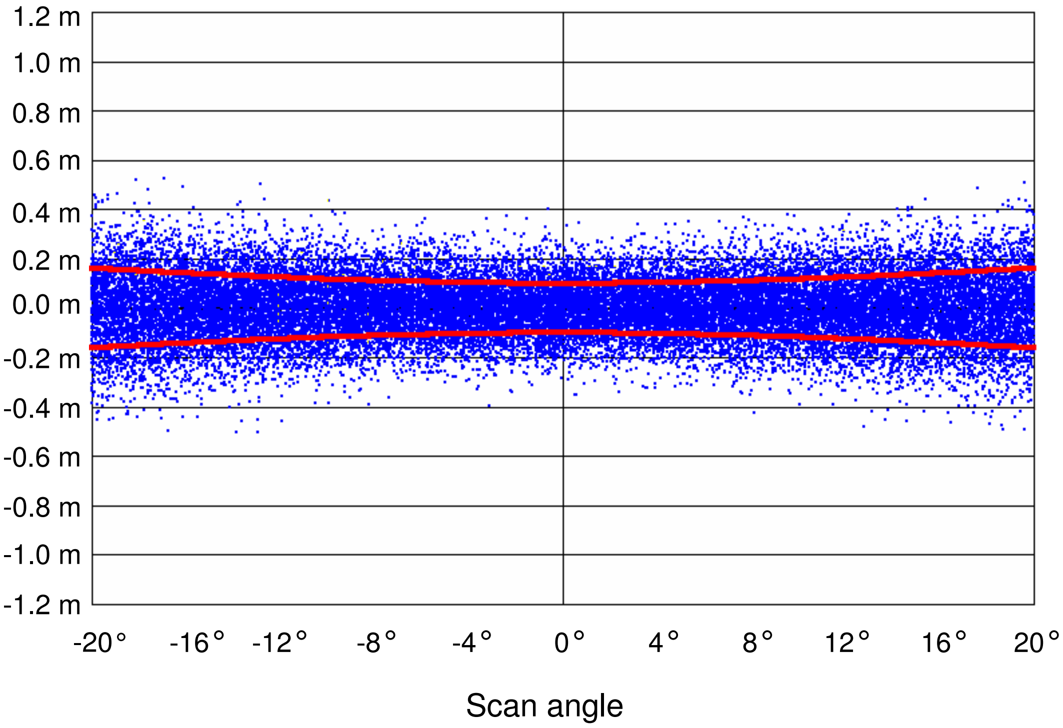
σ_{max} = 0.134 m

σ_{min} = 0.133 m

σ_{mean} = 0.133 m

Stochastic Model – Example AGL 1000

Errors in Northing (In-Flight)



Empirical

max. ε = 0.703 m

min. ε = -0.509 m

mean ε = 0.000 m

rms ε = 0.124 m

$-\sigma \leq \varepsilon \leq \sigma$ = 68.5 %

Theoretical

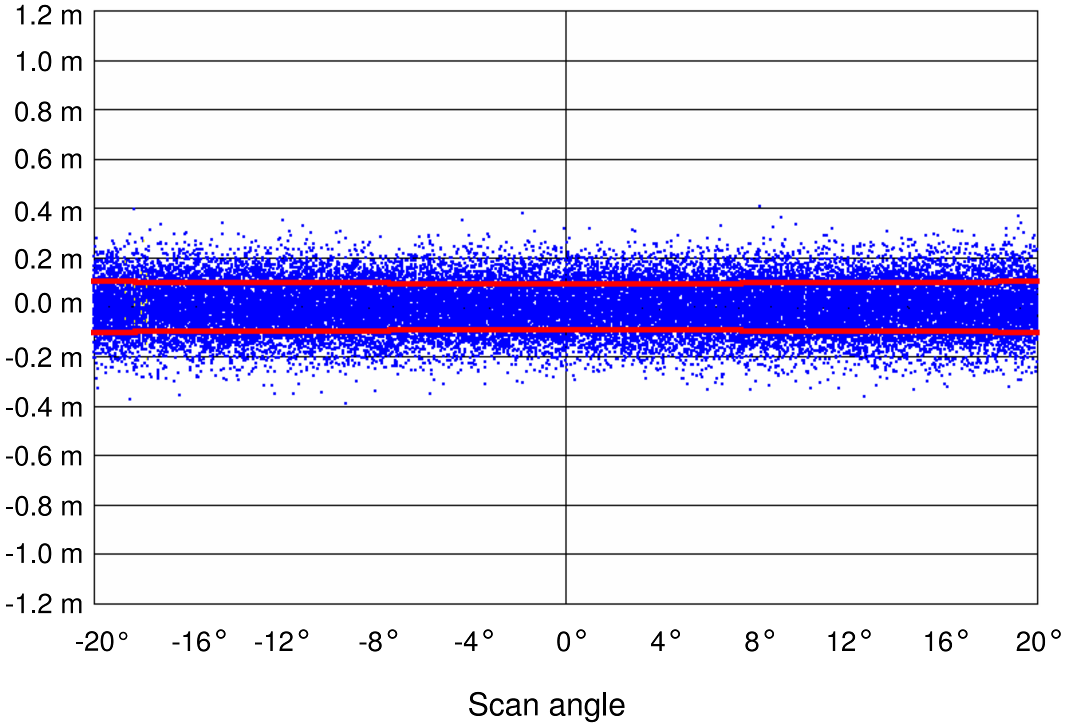
σ_{max} = 0.162 m

σ_{min} = 0.101 m

σ_{mean} = 0.124 m

Stochastic Model – Example AGL 1000

Errors in Height



Empirical

max. ε = 0.409 m

min. ε = -0.390 m

mean ε = 0.000 m

rms ε = 0.097 m

$-\sigma \le \varepsilon \le \sigma$ = 68.5 %

Theoretical

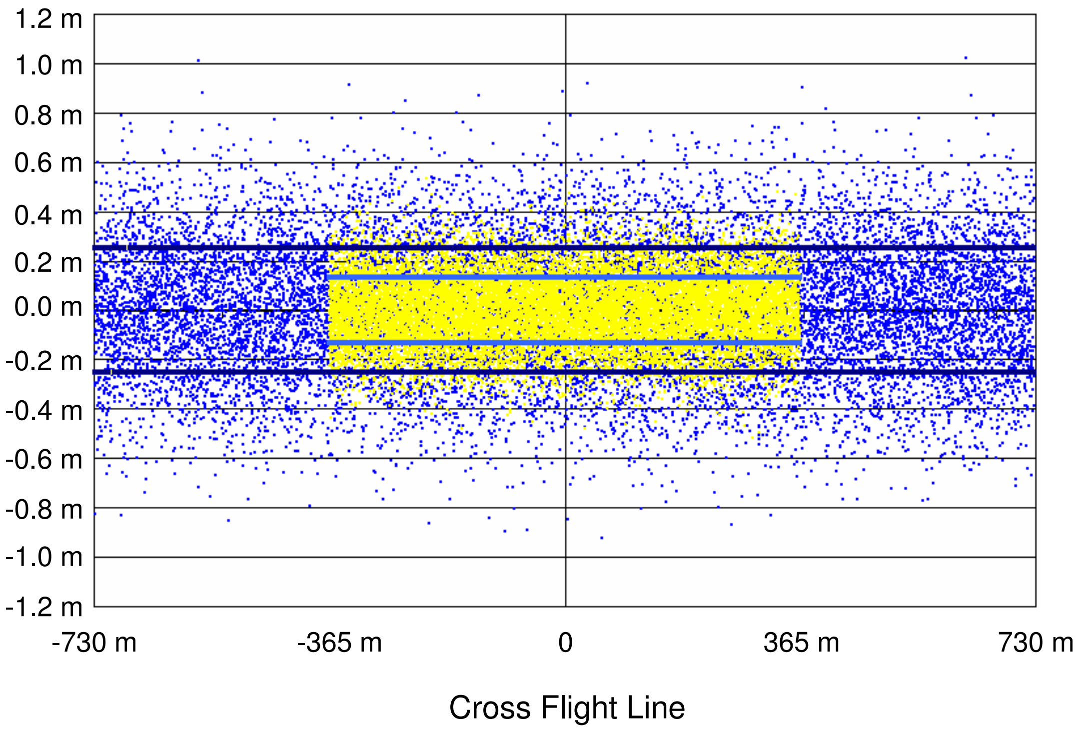
σ_{max} = 0.103 m

σ_{min} = 0.094 m

σ_{mean} = 0.097 m

Stochastic Model – Example

Errors in Easting (Cross-Flight)



For AGL 1000 m

σ_{max} = **0.134** m

σ_{min} = **0.133** m

σ_{mean} = **0.133** m

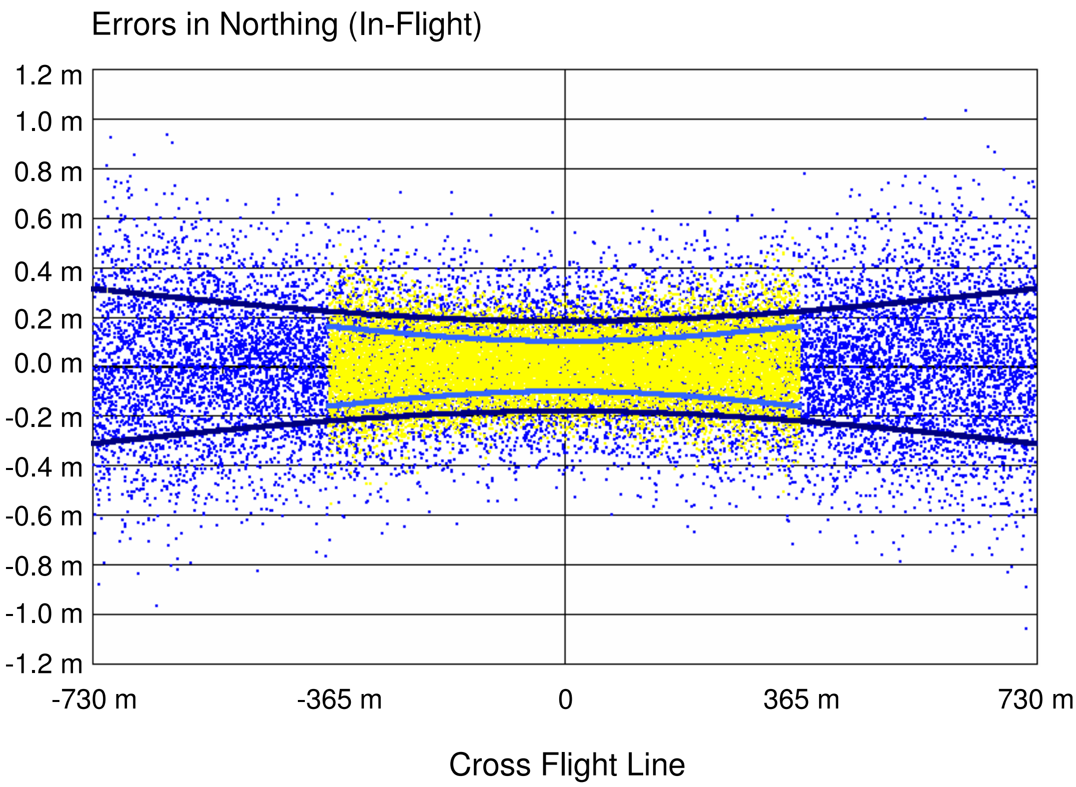
For AGL 2000 m

σ_{max} = **0.252** m

σ_{min} = **0.252** m

σ_{mean} = **0.252** m

Stochastic Model – Example



For AGL 1000 m

σ_{max} = **0.162** m

σ_{min} = **0.101** m

σ_{mean} = **0.122** m

For AGL 2000 m

σ_{max} = **0.312** m

σ_{min} = **0.182** m

σ_{mean} = **0.229** m

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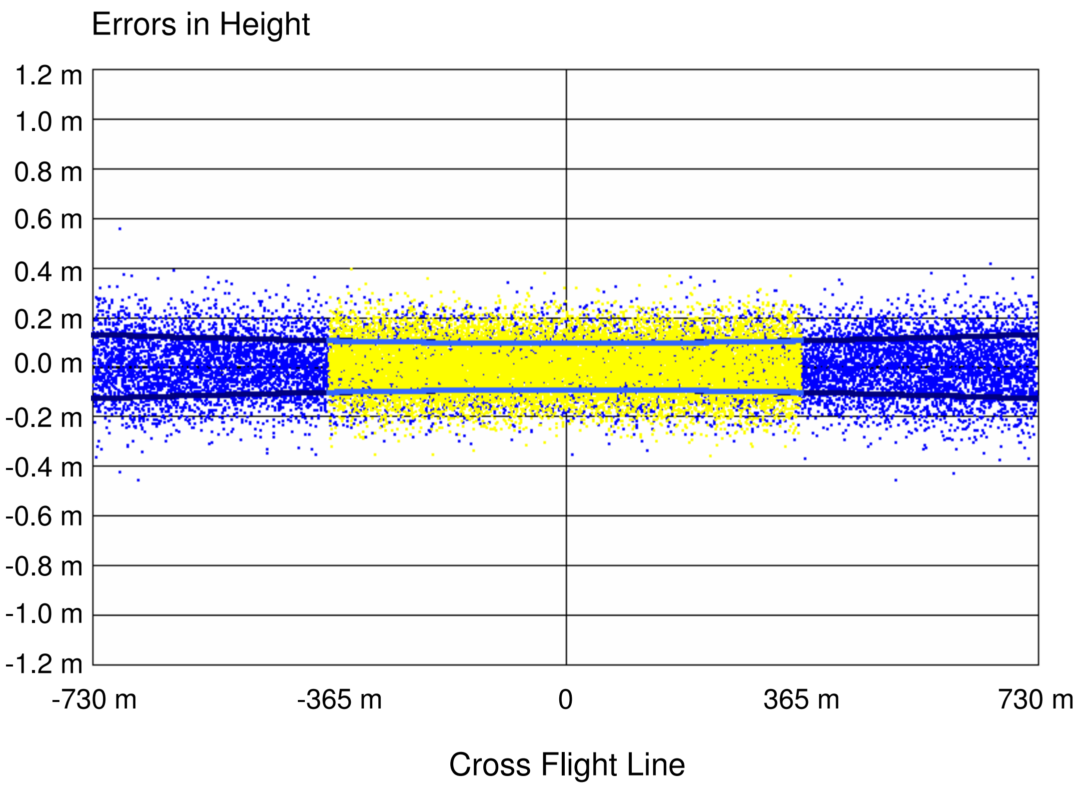
Stochastic Model – Example



For AGL 1000 m

σ_{max} = **0.103** m

σ_{min} = **0.094** m

σ_{mean} = **0.097** m

For AGL 2000 m

σ_{max} = **0.129** m

σ_{min} = **0.094** m

σ_{mean} = **0.107** m

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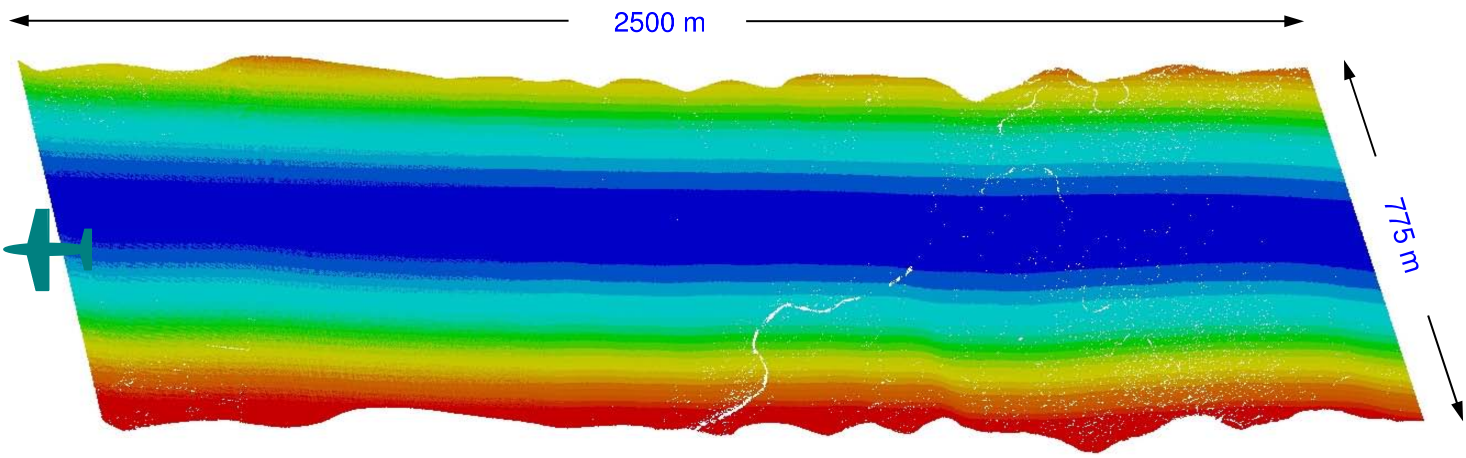
ALM Precision Map



e.g. height standard deviations for the individual laser points

Laser Point Computation – Functional Model (Basic)

Observations

GPS antenna center position $\mathbf{X}_{PC}^{ECEF}$

IMU roll, pitch, heading $\eta, \chi, \alpha \Rightarrow \mathbf{R}_{IBF}^{LGF}$

ALS scan angle, laser range $\Theta, r \Rightarrow \mathbf{X}_{P}^{SBF}$

✓

Mounting parameters

ALS-IMU relative orientation $E_X, E_Y, E_Z \Rightarrow \mathbf{R}_{SBF}^{IBF}$

ALS-Position center eccentricities $\mathbf{X}_{PC}^{SBF}$

?

Corrections

Observation corrections $\Delta r, \Delta\Theta, \Delta s$

Lab (Manufacturer) Calibration

ALS sensor/observation parameters

Realization of sensor body frame (SBF)

Laser range offsets

Scanner offset

Scanner scale factor

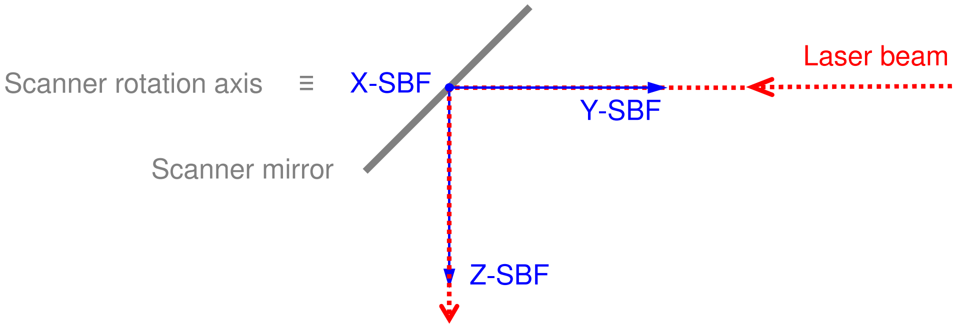
...

ALS-IMU relative orientation

Orientation

Eccentricity

Lab Calibration – Sensor Body Frame

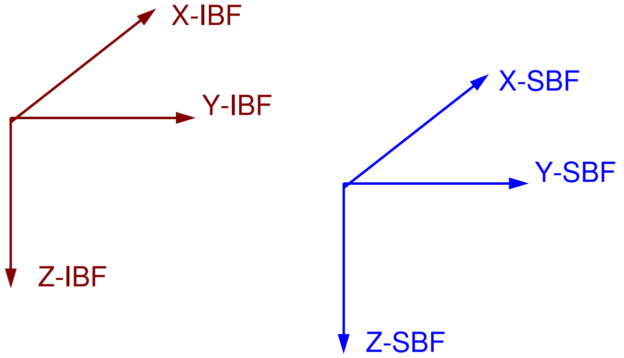


X-SBF
Physically realized by the scanner rotation axis
Points into flight direction

Y-SBF
Physically realized by the incoming laser beam
Incoming laser beam lies in a plane which is perpendicular to X-SBF
Points to the right side of the aircraft

Z-SBF
Physically realized by the outgoing laser beam at zero scan angle
To be perpendicular to XY-plane (completes right-hand cartesian system)
Points down

Lab Calibration – ALS IMU Relative Orientation



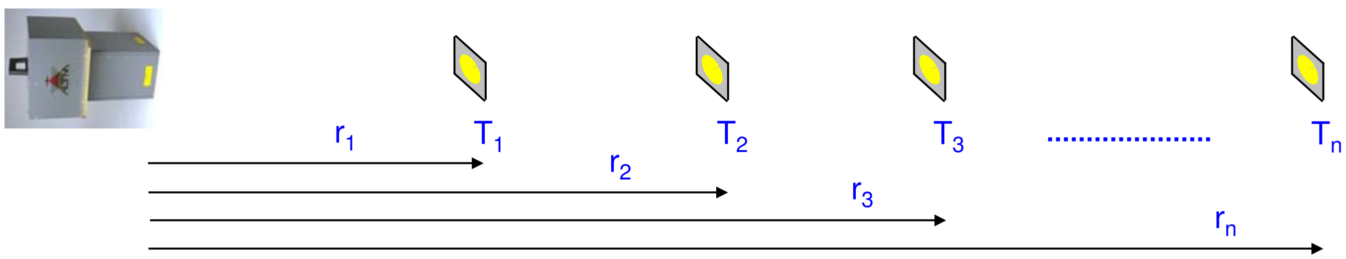
X-SBF (scanner rotation axis) to be parallel to X-IBF (roll axis)

- Scanner angle Θ is equivalent to roll angle η

Y-SBF to be parallel to Y-IBF (pitch axis)

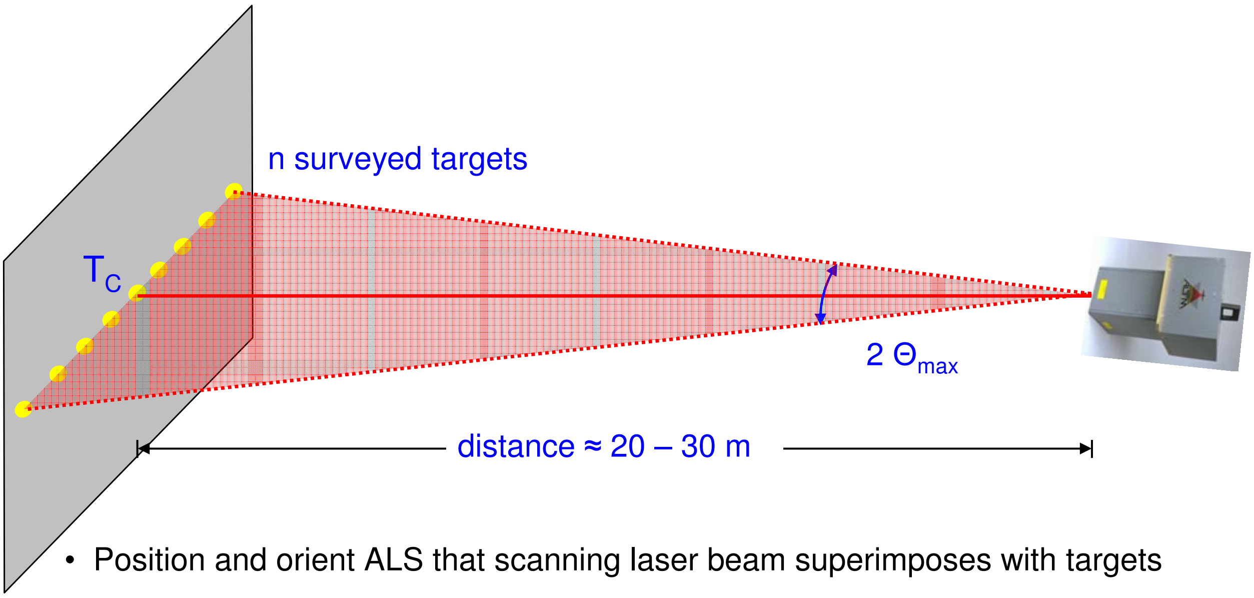
- Scanner angle Θ = 0 corresponds to roll η = 0 and pitch χ = 0
- Permits scanning about local vertical

Lab Calibration – Laser Range Offset



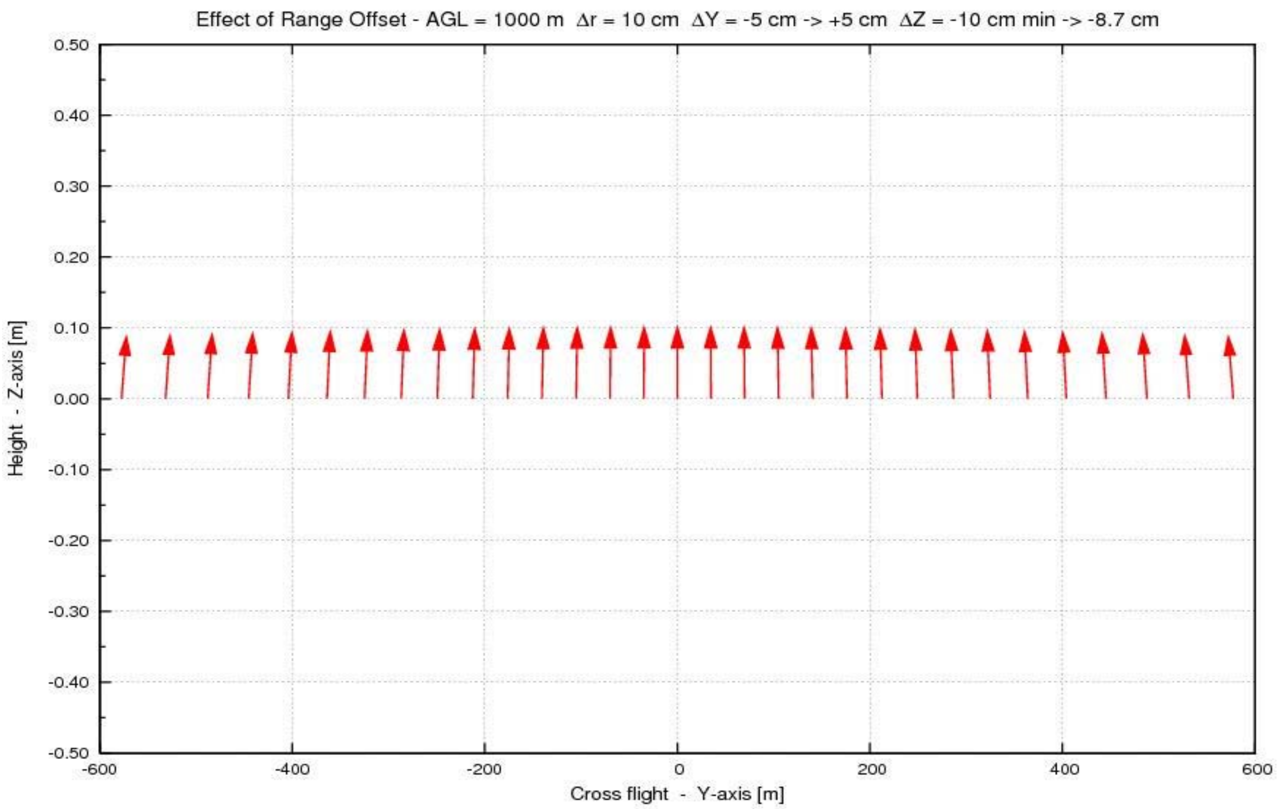
- Place flat targets (T_1 … T_n) at several accurately measured distances (r_1 … r_n)
- Measure a large number of ranges to each of the targets
- Compute the range offset from differences between “true” and measured ranges

Lab Calibration – Scanner Parameters

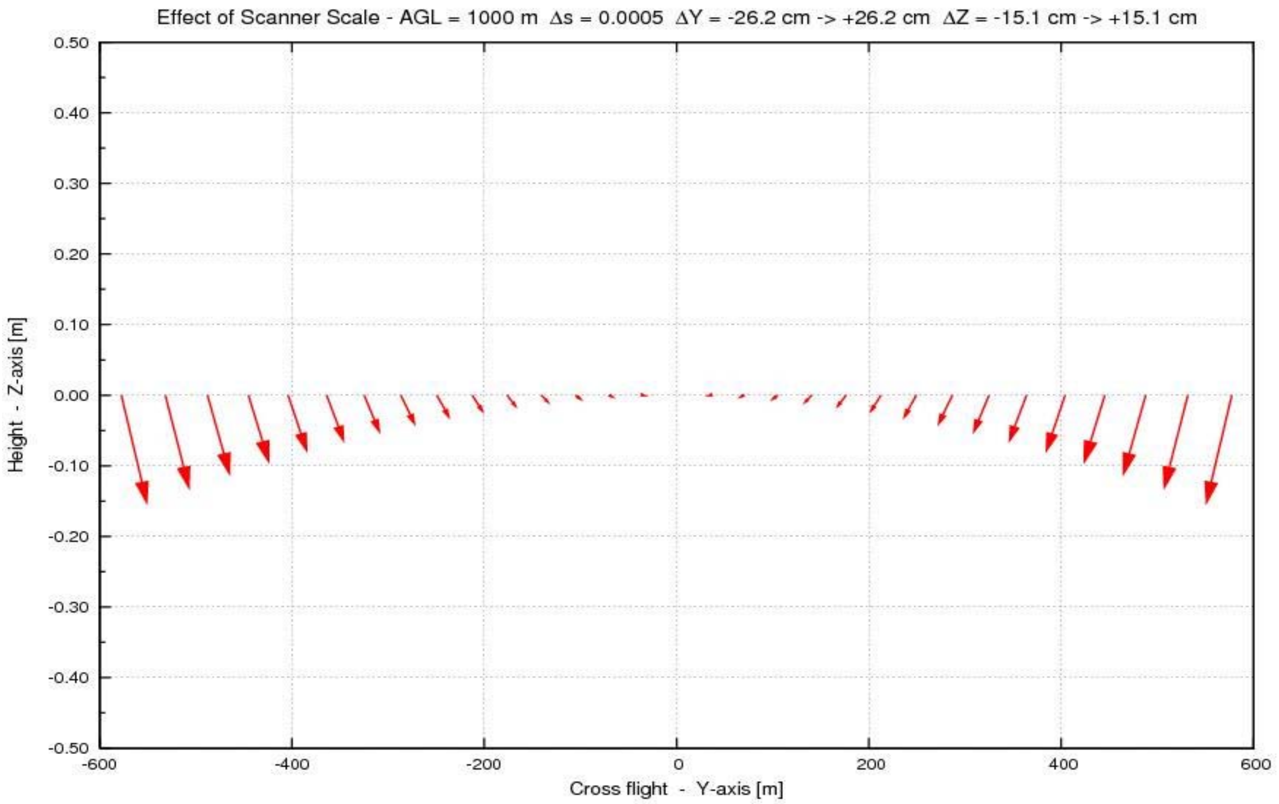


- Position and orient ALS that scanning laser beam superimposes with targets
- Set scan angle = 0 and position beam at center target T_C
- Measure large number of scan angles for the n targets
- Compute scan angle offset and scale from differences between “true” and measured scan angles

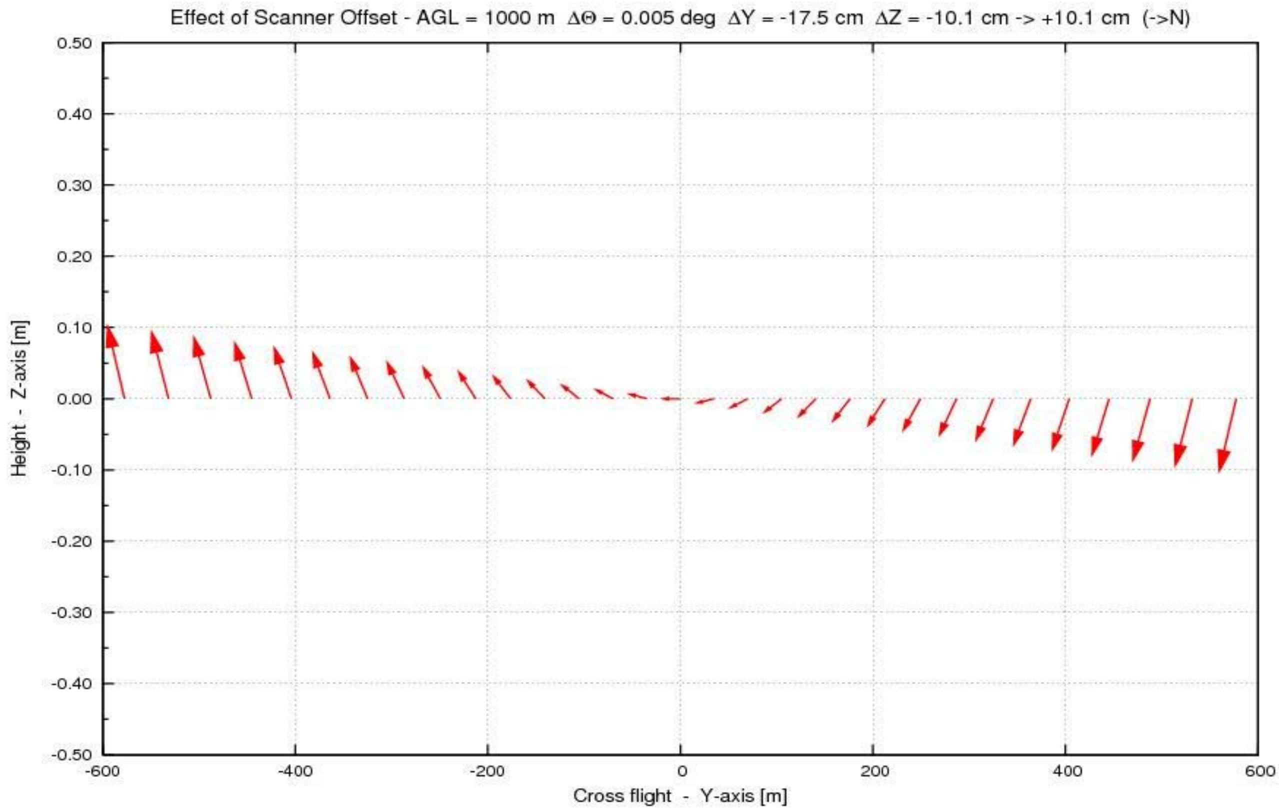
Laser Range Offset – Example



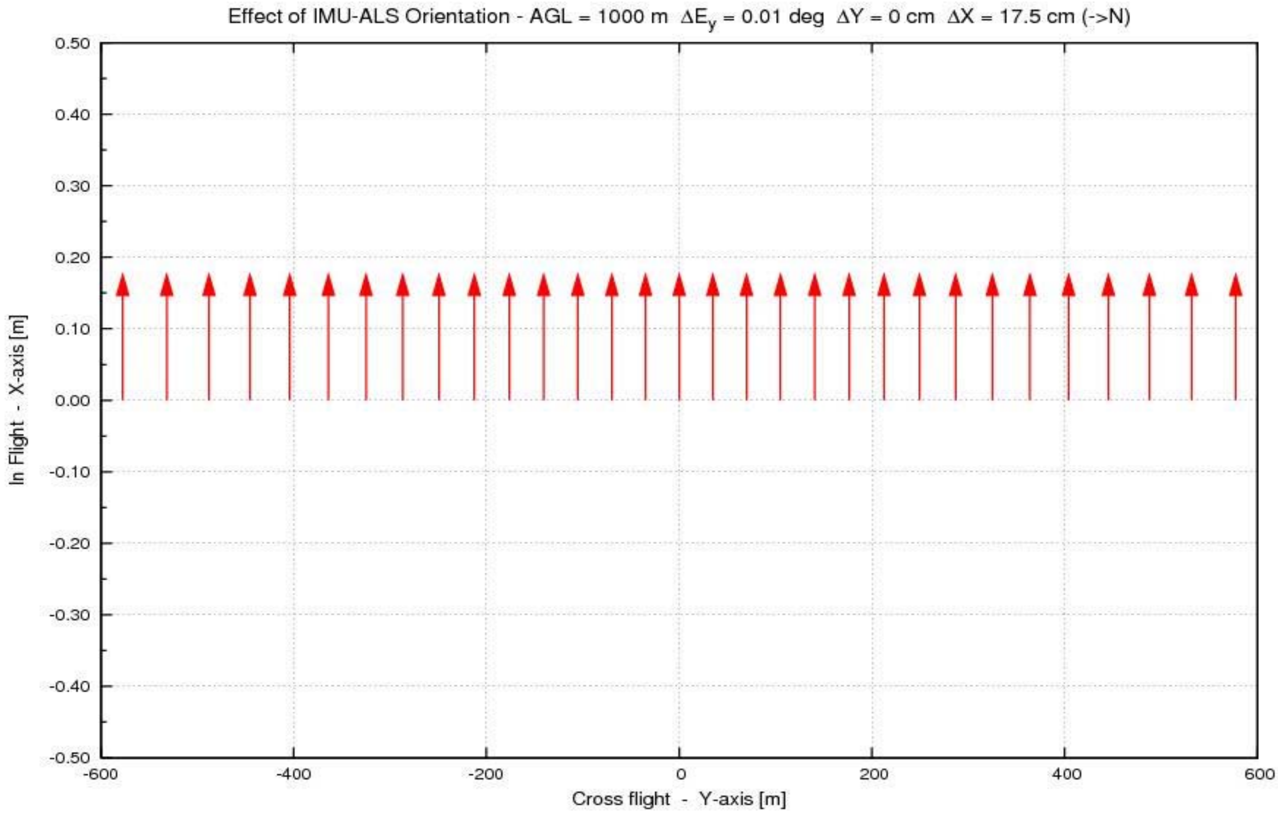
Scan Angle Scale Factor – Example



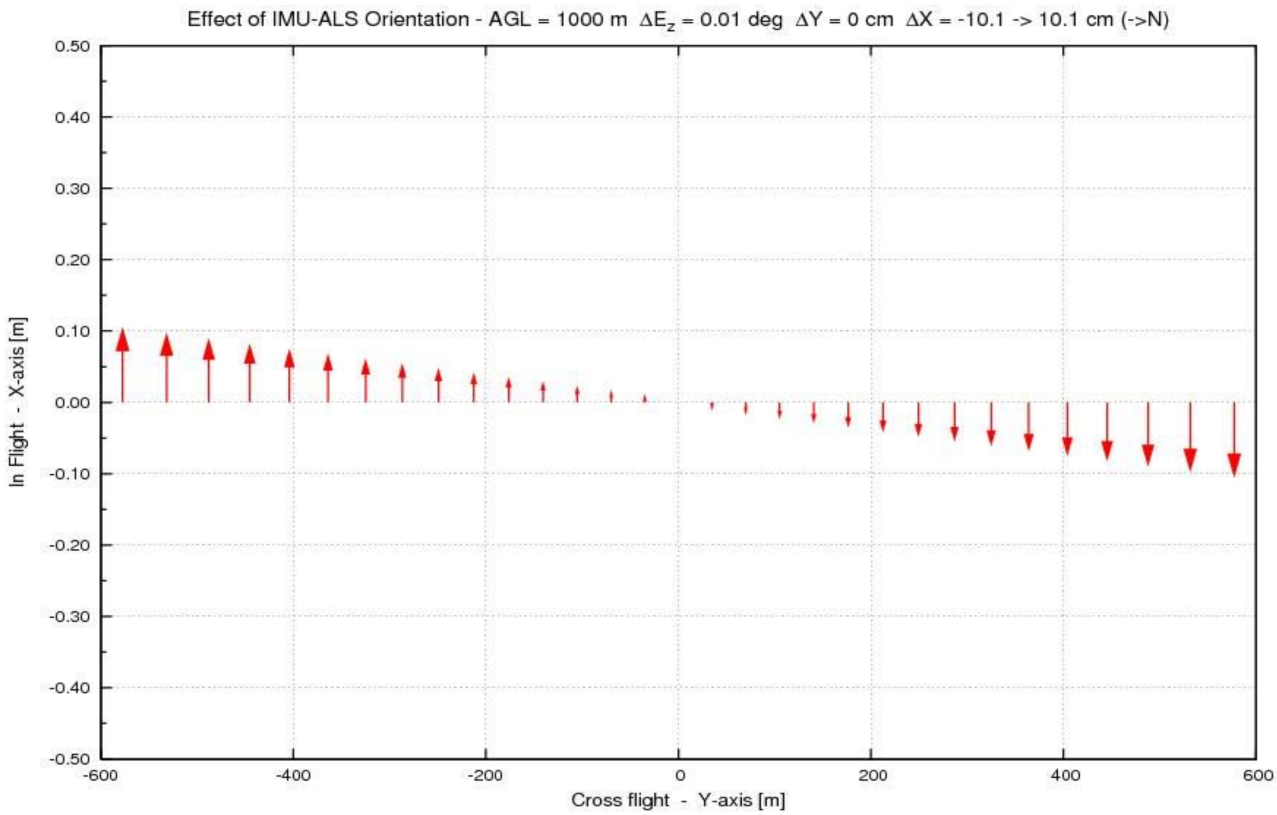
Scan Angle Offset – Example



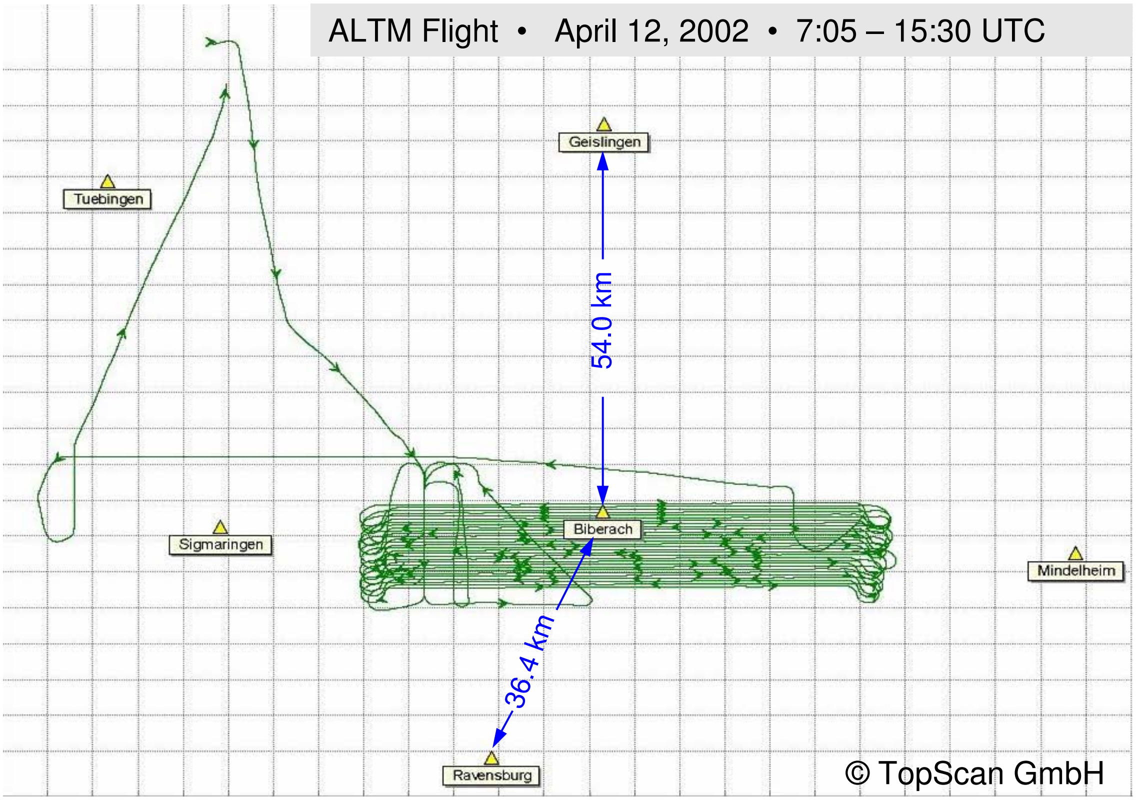
IMU-ALS Relative Orientation E_Y – Example



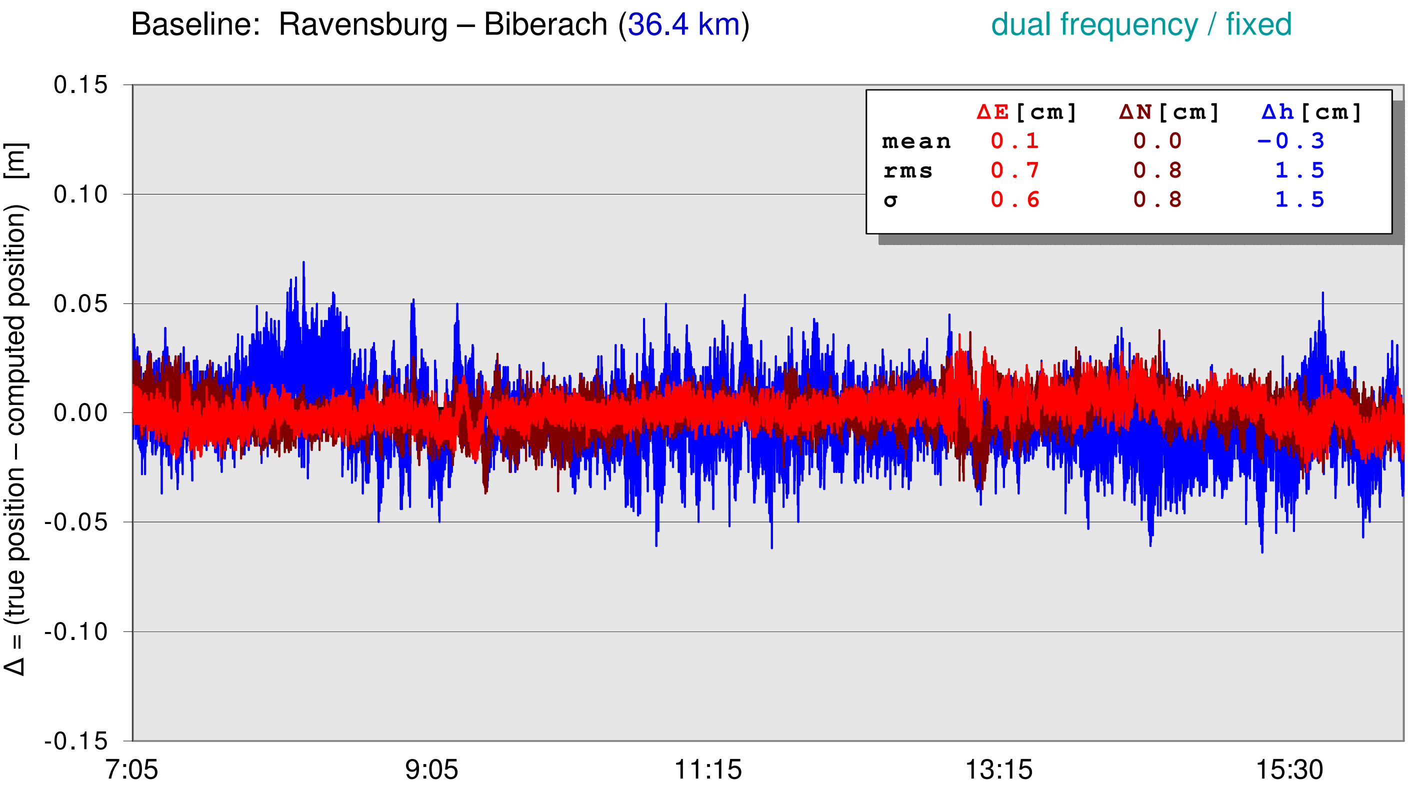
IMU-ALS Relative Orientation E_Z – Example



Relative Kinematic Positioning – Empirical Accuracy Analysis

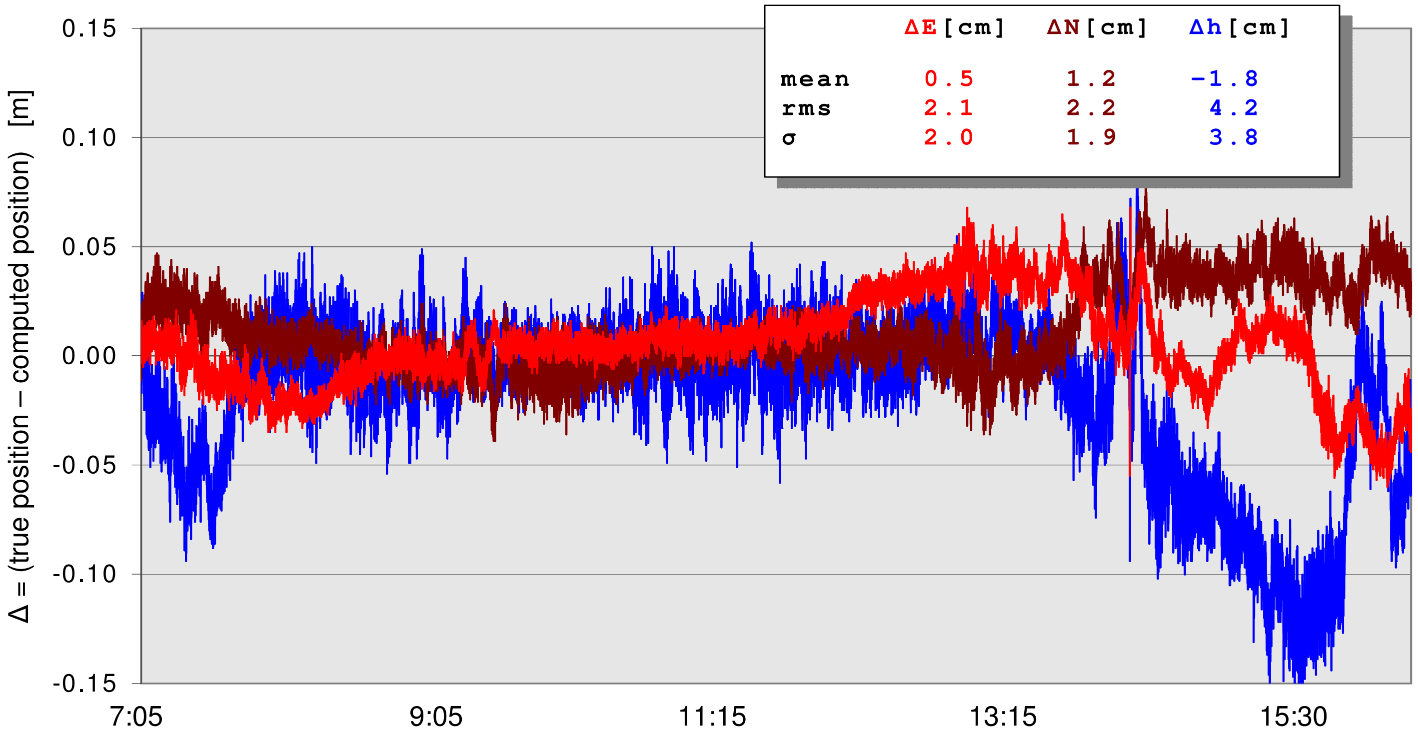


Relative Kinematic Positioning – Least Squares Solution



Relative Kinematic Positioning – KFS Solution

Baseline: Ravensburg – Biberach (36.4 km) dual frequency



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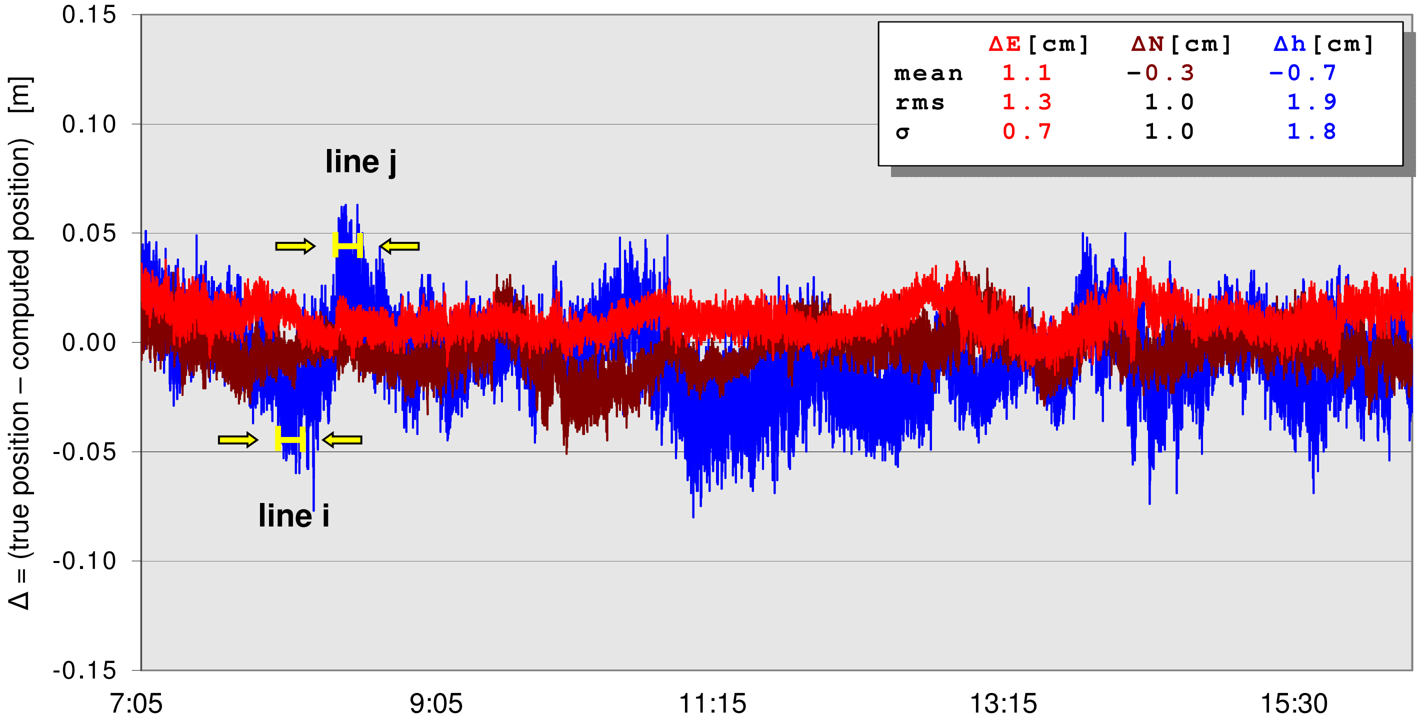
Relative Kinematic Positioning – Least Squares Solution

Baseline: Geislingen – Biberach (54.0 km) dual frequency / float



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Laser Point Computation – Geo-Referencing

$$\mathbf{X}_P^{ECEF} = \mathbf{X}_{PC}^{ECEF} + \Delta\mathbf{X}_{PC}^{ECEF} + \mathbf{R}_{LGF}^{ECEF}\cdot\mathbf{R}_{IBF}^{LGF}\cdot\Delta\mathbf{R}_{IBF}^{LGF}\cdot\mathbf{R}_{SBF,Lab}^{IBF}\cdot\Delta\mathbf{R}_{SBF}^{IBF}\cdot\left(\mathbf{X}_P^{SBF} - \mathbf{X}_{PC,Lab}^{SBF} - \Delta\mathbf{X}_{PC}^{SBF}\right)$$

Sensor

Observations

- GPS antenna center position — $\mathbf{X}_{AC,obs}^{ECEF}$
- IMU roll, pitch, heading — $\eta, \chi, \alpha \Rightarrow \mathbf{R}_{IBF,obs}^{LGF}$
- Laser point in SBF — $\mathbf{X}_P^{SBF}$

Mounting parameters

- ALS-IMU relative orientation — $E_{X,Lab}, E_{Y,Lab}, E_{Z,Lab} \Rightarrow \mathbf{R}_{SBF,Lab}^{IBF}$
- ALS-Position eccentricities — $\mathbf{X}_{PC,Lab}^{SBF}$

Corrections

- GPS position corrections — $\Delta\mathbf{X}_{PC}^{ECEF}$
- IMU attitude corrections — $\Delta\eta, \Delta\chi, \Delta\alpha \Rightarrow \Delta\mathbf{R}_{IBF}^{LGF}$
- ALS-IMU orientation corrections — $\Delta E_X, \Delta E_Y, \Delta E_Z \Rightarrow \Delta\mathbf{R}_{SBF}^{IBF}$
- Eccentricities corrections — $\Delta\mathbf{X}_{PC}^{SBF}$

Laser Point Computation – Sensor Model

$$\mathbf{X}_P^{SBF} = \mathbf{R}(\Theta_P)\cdot\begin{bmatrix}0 & 0 & r_P\end{bmatrix}^T$$

with:

$$r_P = r_{obs} + \Delta r_{Lab} + \Delta r$$

$$\Theta_P = \Theta_{obs}\left(s_{Lab} + \Delta s\right) + \Delta\Theta_{Lab} + \Delta\Theta$$

Observations

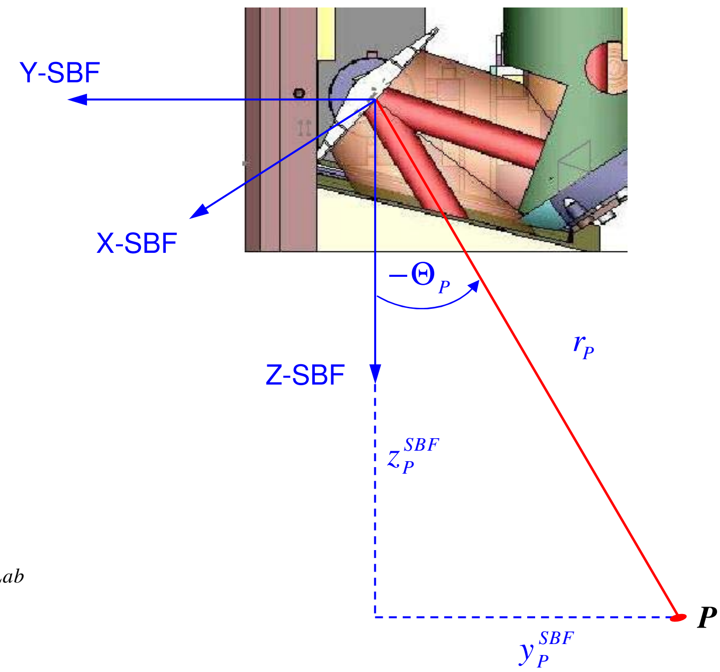
- Laser range — r_{obs}
- Scan-angle — Θ_{obs}

Calibration parameters

- Laser range offset — Δr_{Lab}
- Scan-angle offset and scale factor — $\Delta\Theta_{Lab}, s_{Lab}$

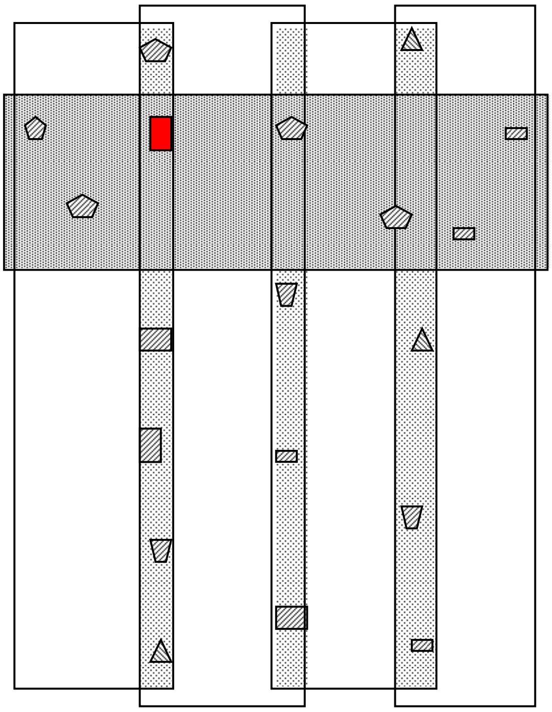
Corrections

- Laser range offset — Δr
- Scan-angle offset and scale factor — $\Delta\Theta, \Delta s$



SBF = Sensor Body Frame

LP Block Adjustment – Approach



Analogy to photogrammetric block adjustment

- Use of planes as tie and control features
- Determine a set of corrections for observations and instrument parameters by minimizing the weighted square sum of the observation residuals

Requires:

⇨ Mathematical sensor model

⇨ Automated planar surface extraction

⇨ Automated planar surface correspondence

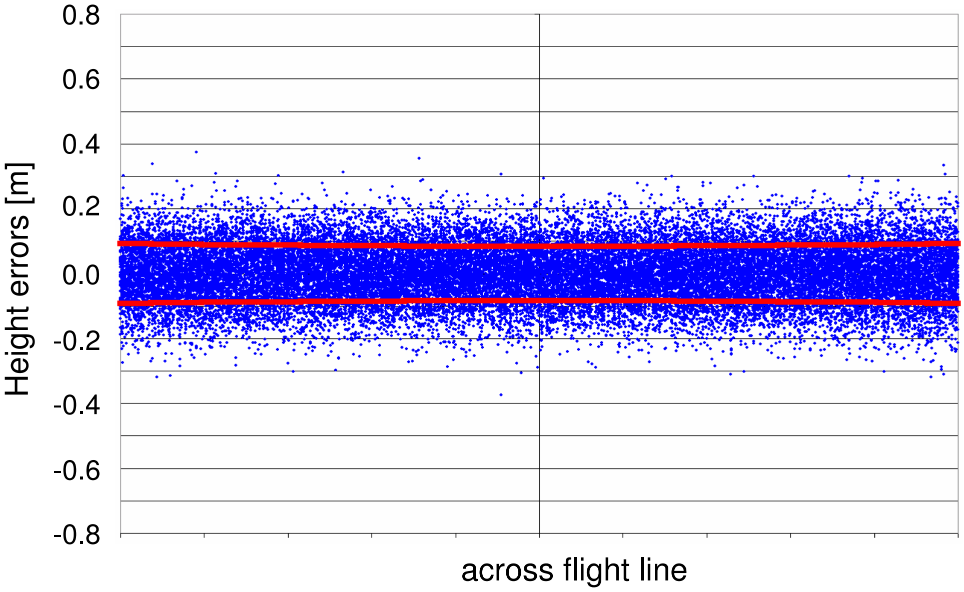
⇨ Parameter estimation (least-squares solution)

LP Block Adjustment – Goal

The goal is a geometrically correct point cloud with known accuracy characteristics

⇨ free of blunders and systematic errors

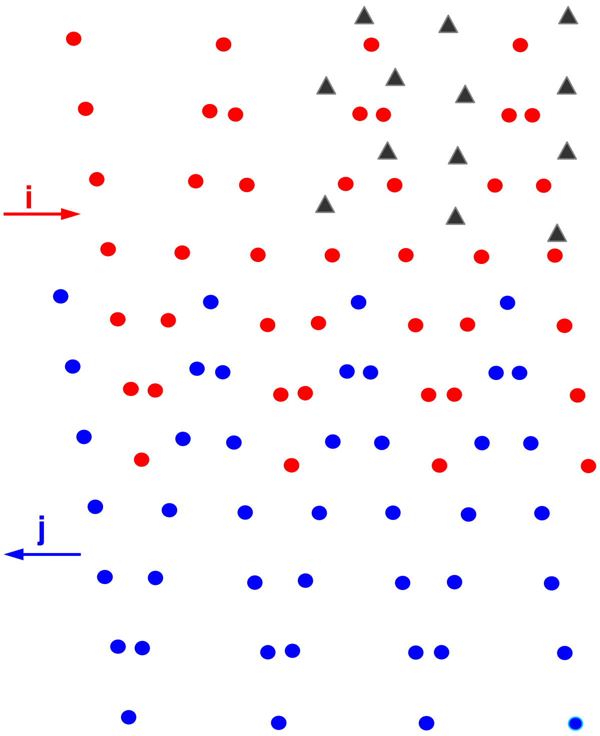
⇨ accuracy given in form of standard deviations for laser point coordinates



only random errors are left

➡ accuracy = precision

Correspondence Problem



Actual, true laser footprint is NOT known !

Therefore:

easy

No direct point to point correspondence between

- laser points (●) and control points (▲)
- laser points (●,●) of overlapping flight lines

Thus:

Correspondence via surface, surface-features

Correspondence Problem – Interpolation & Matching

Principle

Matching techniques provide the horizontal position (X,Y) of a corresponding point

Interpolation techniques provide the height (Z) for the corresponding point

Thus, Interpolation & matching provide tie points

Requirements

Matching requires height variations provided by smooth surfaces with surface normal vectors pointing in three independent directions.

Limitations

Occlusion areas

Height jumps (e.g. on buildings)

Vegetation areas

Correspondence Problem – Surface Features

Principle

Analytically modeling surface features

Least squares fitting

Provides tie features (e.g. planes) with quality attributes

Requirements

E.g planar surface patches have to exist in project area

Limitations

Vegetation areas

?

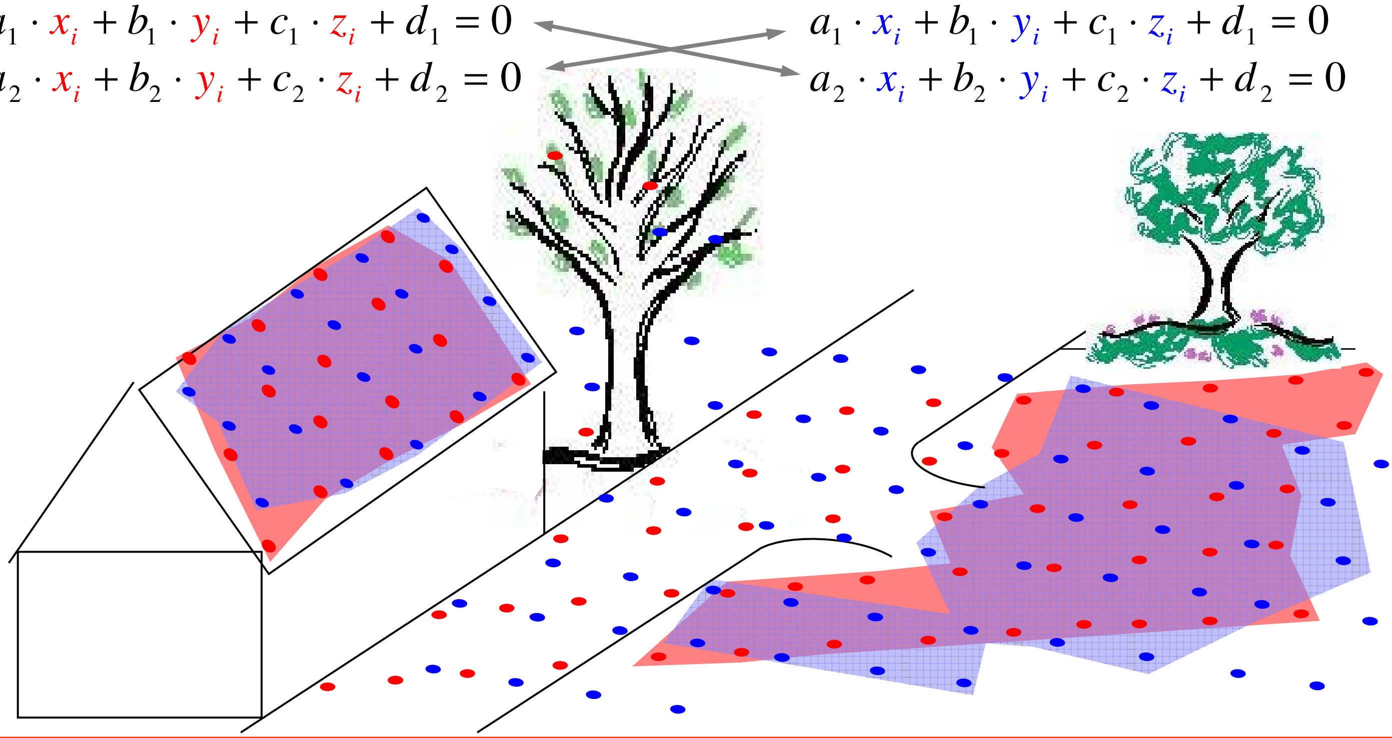
Surface Feature – Plane

Flight Line A

$$a_1 \cdot x_i + b_1 \cdot y_i + c_1 \cdot z_i + d_1 = 0$$
$$a_2 \cdot x_i + b_2 \cdot y_i + c_2 \cdot z_i + d_2 = 0$$

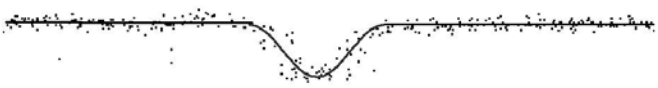
Flight Line B

$$a_1 \cdot x_i + b_1 \cdot y_i + c_1 \cdot z_i + d_1 = 0$$
$$a_2 \cdot x_i + b_2 \cdot y_i + c_2 \cdot z_i + d_2 = 0$$

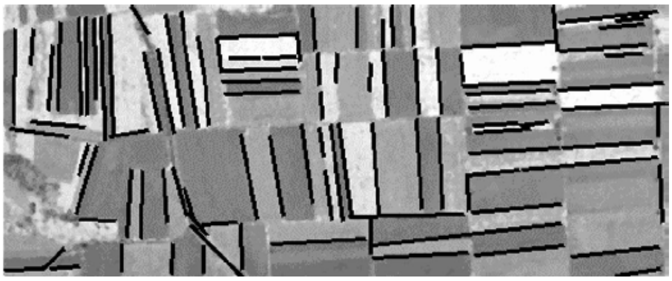


Surface Feature – Ditch

Vosselman G. 2002



Fitting an analytical model for a ditch height profile to the laser points for each flight line.

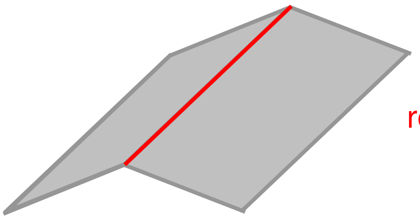


Fitting an analytical model for an intensity edge to the laser point intensity values before applying edge detection.

Surface Feature – Line

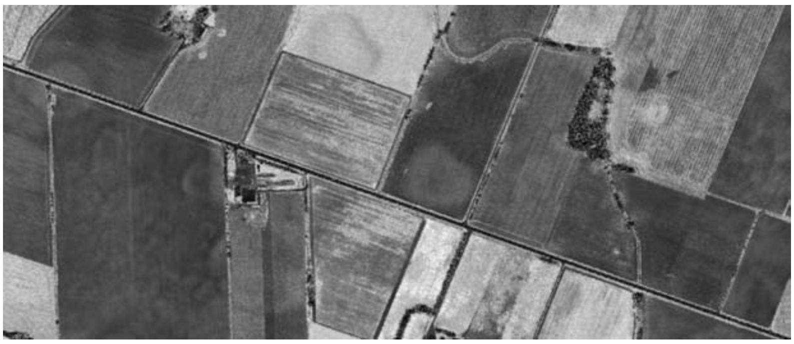
Intersection of laser point planes

- Roof lines are horizontal
- Roof lines are known



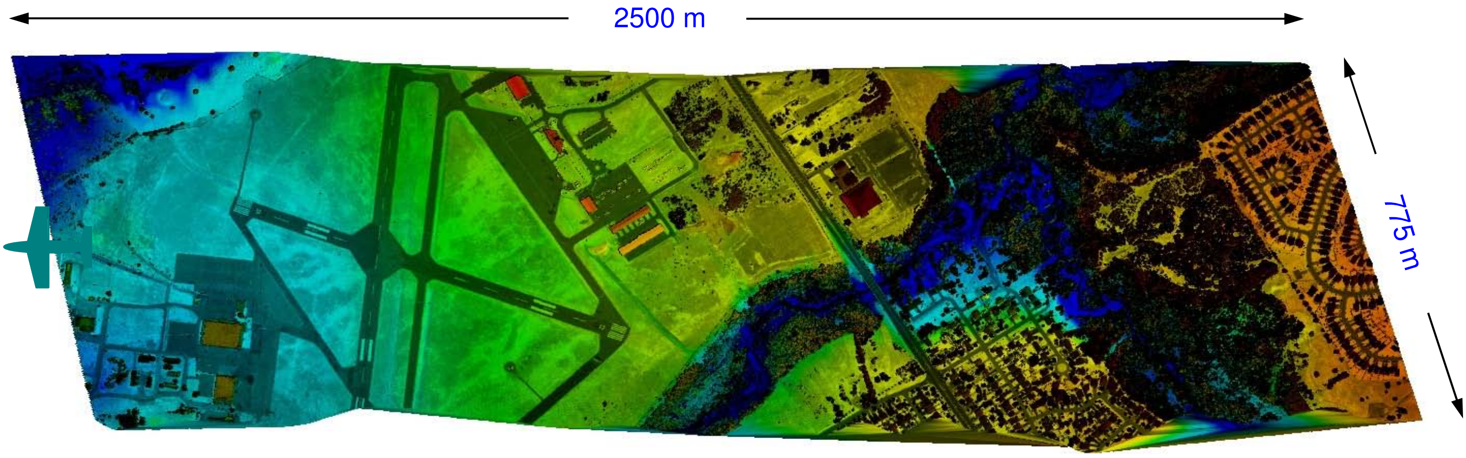
Laser point lines (2D, 3D)

- Lines as tie feature
- Lines as control feature



⇨ Simultaneous laser point and photogrammetric block adjustment

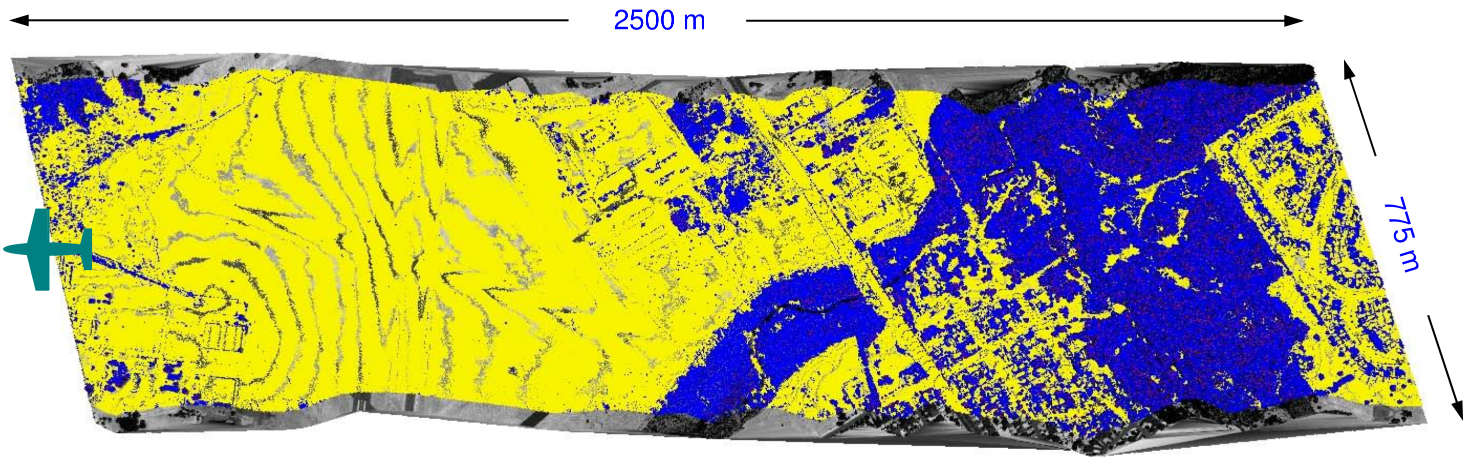
Empirical Studies – Runway Line 25



Flight Parameters

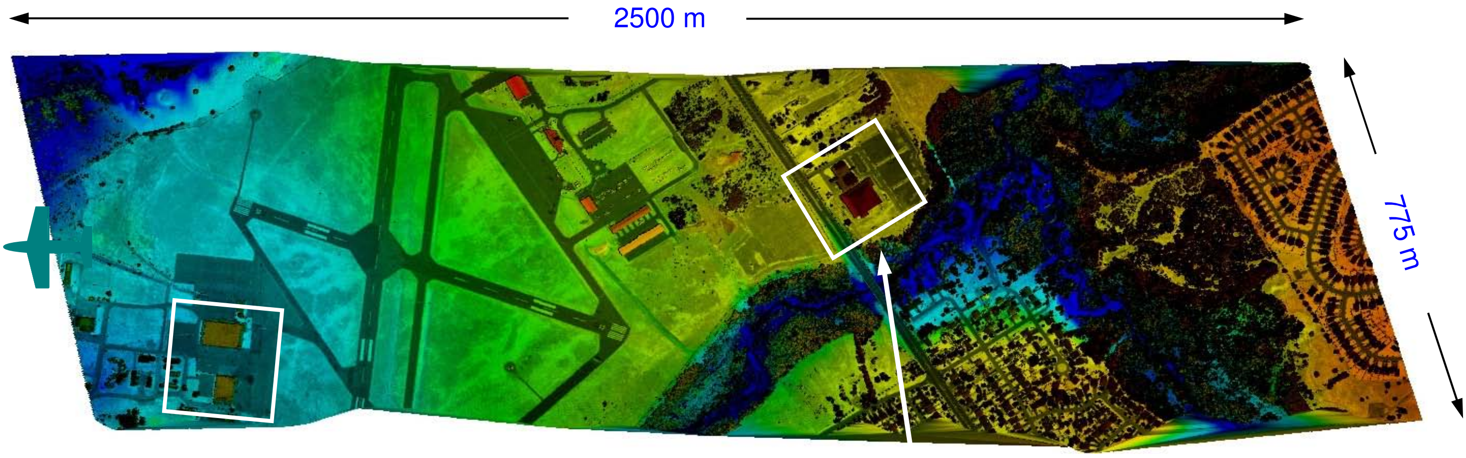
- January 2, 2003
- One flight line approx. 2500 m long (41.24 seconds at 60.6 m/s ground speed)
- Flying height 1100 m above ground
- ALTM 2050 @ 35 KHz laser rep. rate, ± 20 deg scan-angle, 35 Hz scan-frequency

Empirical Studies – Runway Line 25: Plane- vs. None-Plane Points



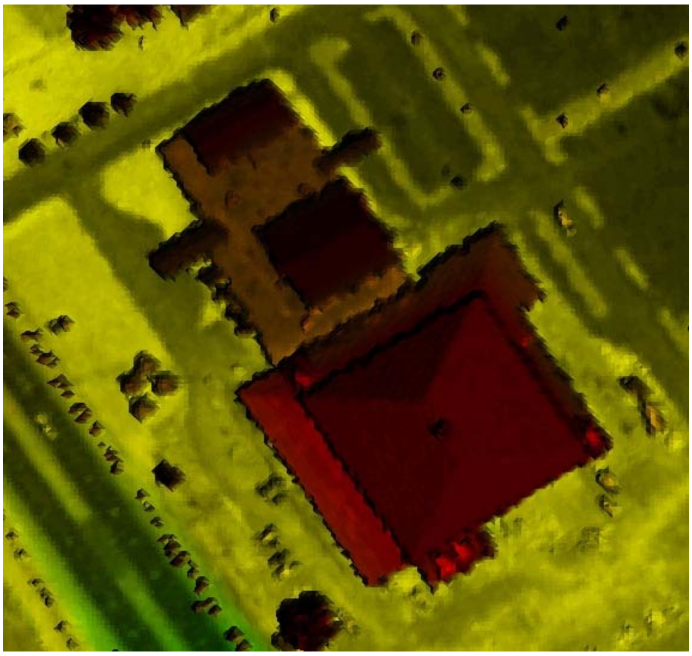
- 997206 laser points total
- 26650 having two returns 2.7 %
- 280852 none plane points 28.2 %
- **689714 plane points 69.1 %**

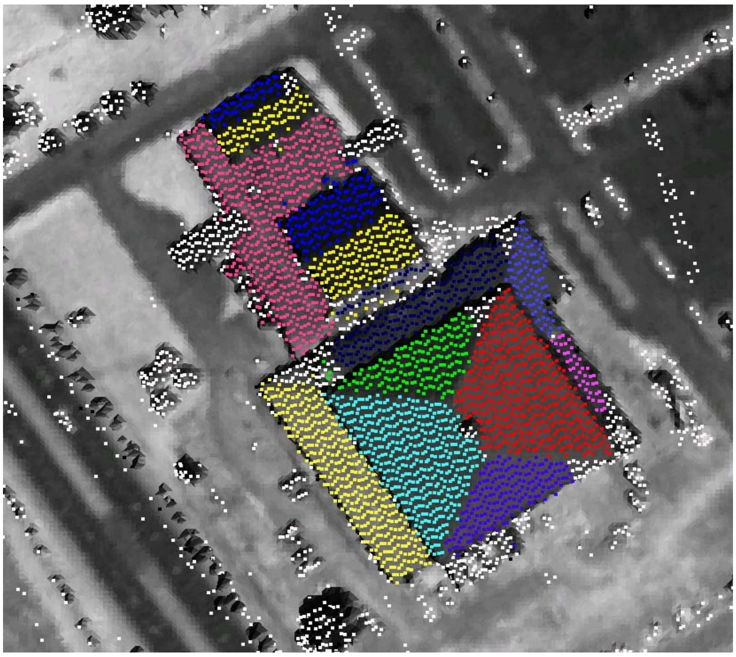
Empirical Studies – Runway Line 25



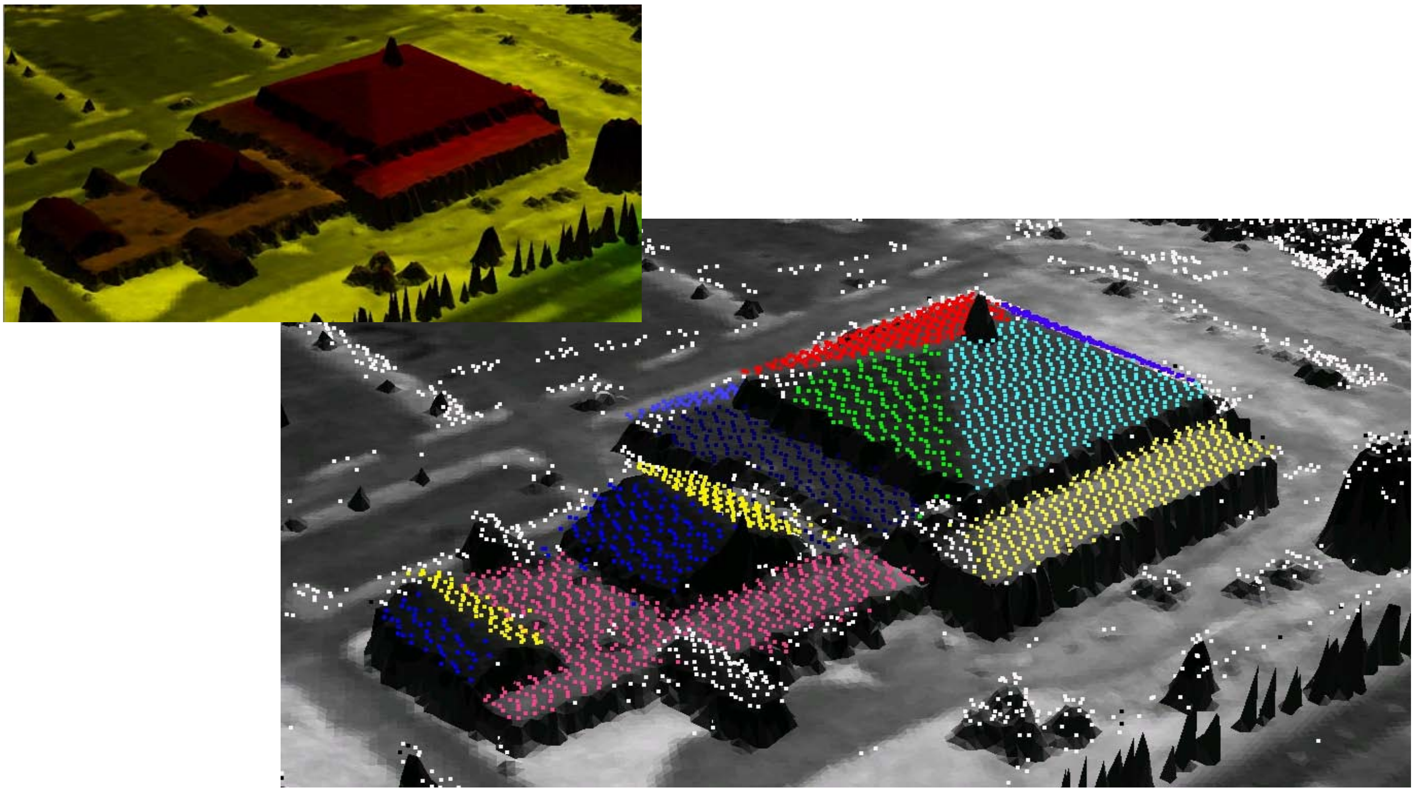
Building 3 Building 1

Empirical Studies – Runway Line 25 Building 1





Empirical Studies – Runway Line 25 Building 1



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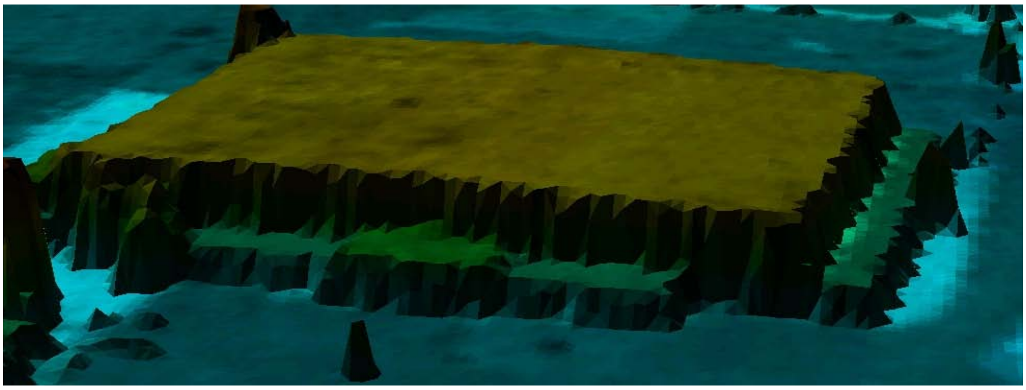
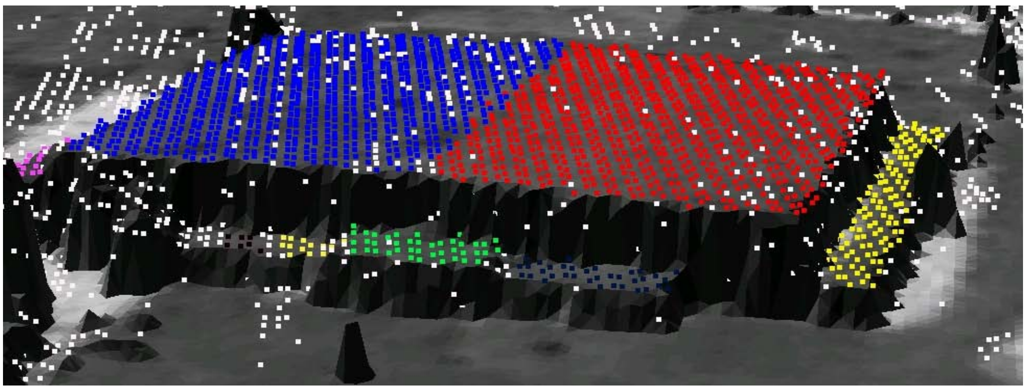
Empirical Studies – Runway Line 25 Building 3





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Least Squares Adjustment – Observation Equation I

Observation equation

$$g_{i,j} = n_{X,j}x_i + n_{Y,j}y_i + n_{Z,j}z_i + d = 0$$

Implicit non-linear model

$$\mathbf{g}(\mathbf{l}+\mathbf{v},\mathbf{x}) = \mathbf{0}$$

Implicit linear model at $(\mathbf{l},\mathbf{x}_i)$

$$\mathbf{g}(\mathbf{l},\mathbf{x}_i) + \left(\frac{\delta \mathbf{g}}{\delta \mathbf{l}}\right)_{|(\mathbf{l},\mathbf{x}_i)} \cdot \mathbf{v} + \left(\frac{\delta \mathbf{g}}{\delta \mathbf{x}}\right)_{|(\mathbf{l},\mathbf{x}_i)} \cdot \mathbf{x} = \mathbf{0} \quad \text{or} \quad \mathbf{g} + \mathbf{D}\cdot\mathbf{v} + \mathbf{A}\cdot\mathbf{x} = \mathbf{0}$$

where:

- $\mathbf{l}$ Given observations
- $\mathbf{v}$ Unknown residuals of the observations, with $\mathbf{v} \sim N\left(\mathbf{0},\mathbf{C}_{vv}\right)$
- $\mathbf{x}$ Unknown parameters

Least Squares Adjustment – Observation Equation II

Observations

GPS antenna center position	$\mathbf{X}_{AC,obs}^{ECEF}$
IMU roll, pitch, heading	$\eta_{obs}, \chi_{obs}, \alpha_{obs}$
ALS scan angle, laser range	Θ_{obs}, r_{obs}
Control plane information	$\Delta\lambda_{PS}, \Delta\varphi_{PS}, d$, or x_{PS}, y_{PS}, z_{PS}

(Pseudo-observations for all unknowns)

Unknowns

Plane parameters	$\Delta\lambda_{PS}, \Delta\varphi_{PS}, d$
GPS antenna eccentricity corrections	$\Delta x_{PC}^{SBF}, \Delta y_{PC}^{SBF}, \Delta z_{PC}^{SBF}$
ALS-IMU relative orientation corrections	$\Delta \boldsymbol{E}_X, \Delta \boldsymbol{E}_Y, \Delta \boldsymbol{E}_Z$
IMU attitude corrections	$\Delta\eta, \Delta\chi, \Delta\alpha$
Position corrections	$\Delta x_{PC}^{ECEF}, \Delta y_{PC}^{ECEF}, \Delta z_{PC}^{ECEF}$
Laser range correction	Δr
Scan angle corrections	$\Delta\Theta, \Delta s$

Least Squares Adjustment – Solution

Best linear unbiased estimate $\hat{\mathbf{x}}$ for : $\mathbf{x}$

$$\mathbf{A}^T \cdot \left(\mathbf{D} \cdot \mathbf{C}_{vv} \cdot \mathbf{D}^T\right)^{-1} \cdot \mathbf{A} \cdot \hat{\mathbf{x}} = \mathbf{A}^T \cdot \left(\mathbf{D} \cdot \mathbf{C}_{vv} \cdot \mathbf{D}^T\right)^{-1} \cdot \mathbf{g}$$

and

$$\mathbf{C}_{\hat{x}\hat{x}} = \left(\mathbf{A}^T \cdot \left(\mathbf{D} \cdot \mathbf{C}_{vv} \cdot \mathbf{D}^T\right)^{-1} \cdot \mathbf{A}\right)^{-1}$$

$$\hat{\mathbf{v}} = \mathbf{C}_{vv} \cdot \mathbf{D}^T \cdot \left(\mathbf{D} \cdot \mathbf{C}_{vv} \cdot \mathbf{D}^T\right)^{-1} \cdot \left(\mathbf{A} \cdot \hat{\mathbf{x}} - \mathbf{g}\right)$$

$$\mathbf{D} \cdot \mathbf{C}_{\hat{v}\hat{v}} \cdot \mathbf{D}^T = \mathbf{D} \cdot \mathbf{C}_{vv} \cdot \mathbf{D}^T - \mathbf{A}^T \cdot \mathbf{C}_{\hat{x}\hat{x}} \cdot \mathbf{A}$$

where

$\mathbf{C}_{\hat{x}\hat{x}}$ Covariance matrix of the estimated parameters

$\hat{\mathbf{v}}$ Estimated residuals of the observations

$\mathbf{C}_{\hat{v}\hat{v}}$ Covariance matrix of the estimated residuals

LP Block Adjustment – Parameter Determinability I

Observation corrections

Scan angle corrections $\Delta\Theta$

Scan angle corrections Δs

Laser range correction Δr

Mounting parameters

~~ALS-IMU orientation ΔE_X~~

ALS-IMU orientation ΔE_Y

ALS-IMU orientation ΔE_Z

Position eccentricity Δx_{PC}^{SBF}

Position eccentricity Δy_{PC}^{SBF}

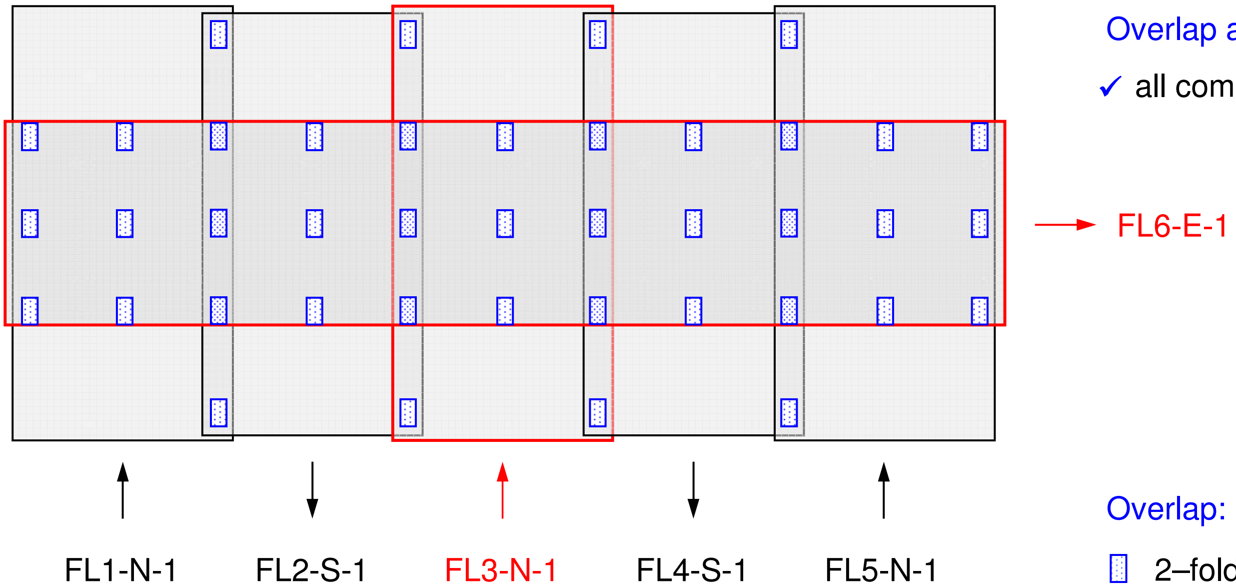
Position eccentricity Δz_{PC}^{SBF}

Geo-Referencing corrections per block

Position correction	Δx_{PC}^{LPF}	~~Attitude correction~~	~~$\Delta\eta$~~
Position correction	Δy_{PC}^{LPF}	~~Attitude correction~~	~~$\Delta\chi$~~
Position correction	Δz_{PC}^{LPF}	~~Attitude correction~~	~~$\Delta\alpha$~~

LP Block Adjustment – Simulation Study I

Block Configuration



Overlap areas:

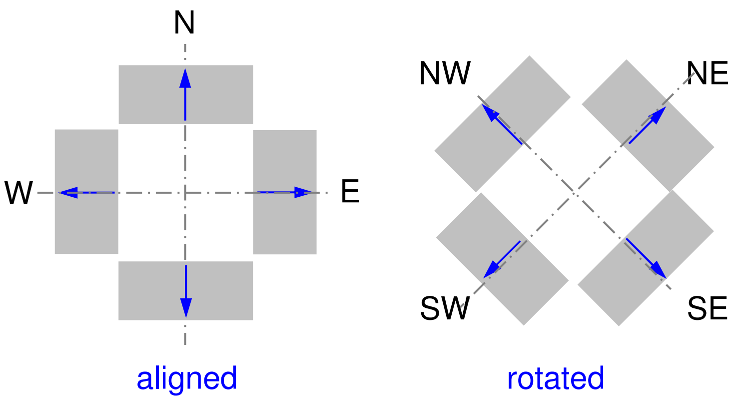
✓ all combinations

Overlap:

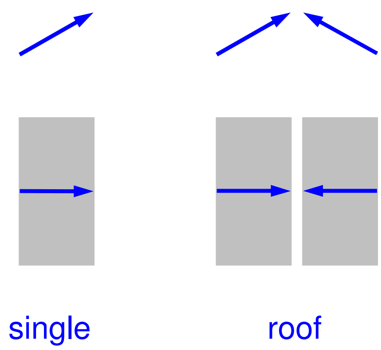
- 2–fold
- 3–fold

LP Block Adjustment – Simulation Study II

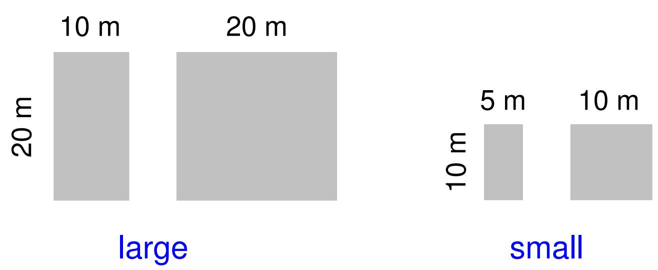
Plane Orientation



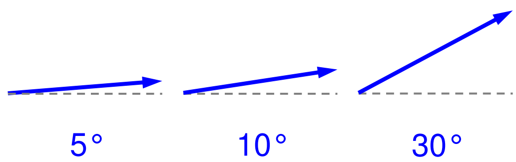
Plane Type



Plane Size



Plane Slope



LP Block Adjustment – Parameter Determinability II

Observation corrections

Scan angle corrections	$\Delta\Theta$	can 'always' be determined
Scan angle corrections	Δs	
Laser range correction	Δr	horizontal **control** plane(s) in the swath center

Mounting parameters

ALS-IMU orientation	ΔE_Y	sloped planes (≥ 5°) with normal vectors parallel to the flight direction
ALS-IMU orientation	ΔE_Z	

⇨ Basically one cross flight line and (a) horizontal control plane(s) located in the swath center allow for the determination of $\Delta\Theta, \Delta s, \Delta r, \Delta E_Y$ and ΔE_Z

LP Block Adjustment – Parameter Determinability III

Observation corrections

Scan angle corrections	$\Delta\Theta$	can 'always' be determined
Scan angle corrections	Δs	

Mounting parameters

ALS-IMU orientation	ΔE_Y	sloped planes (≥ 5°) with normal vectors parallel to the flight direction
ALS-IMU orientation	ΔE_Z	

Geo-Referencing corrections per block

Position correction	Δx_{PC}^{LPF}	sloped **control** planes (> 10°) with normal vectors perpendicular to each other
Position correction	Δy_{PC}^{LPF}	
Position correction	Δz_{PC}^{LPF}	horizontal **control** plane(s) in the swath center

LP Block Adjustment – Parameter Determinability IV

Observation corrections

Scan angle corrections	$\Delta\Theta$	can 'always' be determined
Scan angle corrections	Δs	

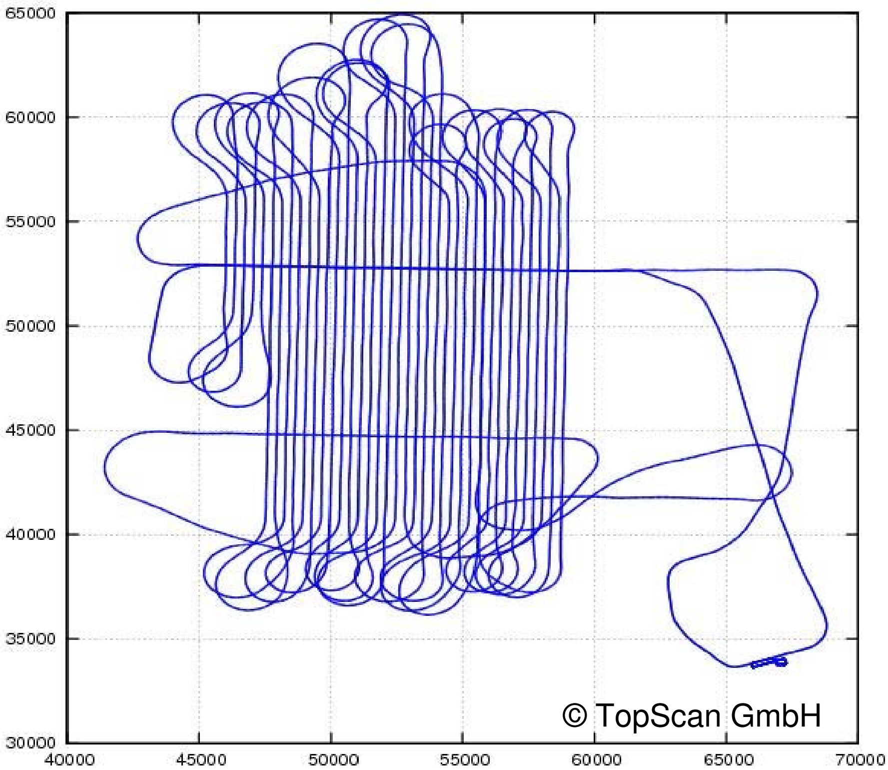
Mounting parameters

ALS-IMU orientation	$\Delta \boldsymbol{E}_Y$	sloped planes (≥ 5°) with normal vectors parallel to the flight direction
ALS-IMU orientation	$\Delta \boldsymbol{E}_Z$	
Position eccentricity	Δx_{PC}^{SBF}	two different flying heights required, but…
Position eccentricity	Δy_{PC}^{SBF}	sloped planes (> 5°) with normal vector perpendicular to flight direction

Geo-Referencing corrections per block

Position correction	Δx_{PC}^{LPF}	sloped **control** planes (> 10°) with normal vectors perpendicular to each other
Position correction	Δy_{PC}^{LPF}	
Position correction	Δz_{PC}^{LPF}	horizontal **control** plane(s) in the swath center

Empirical Test 'Rheine' – Flight Path



Flight Parameters

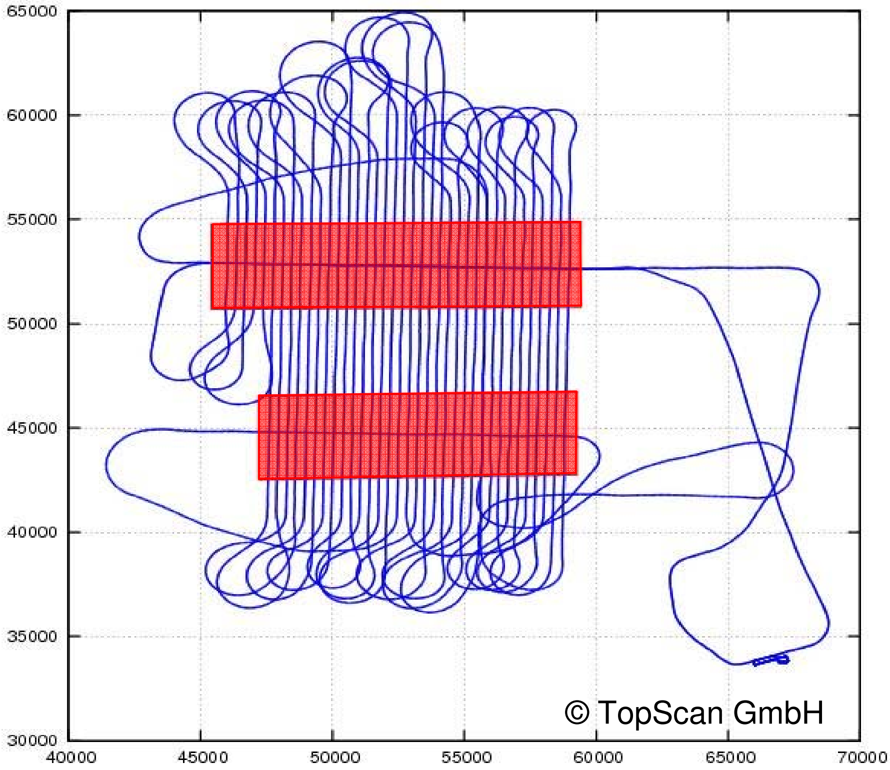
- November 5, 2003
- 4 h 25 m flight duration
- 950 m flying height AGL
- ALTM 1225

  25 KHz laser rep. rate

  25 Hz scan-frequency

  ± 20 deg scan-angle

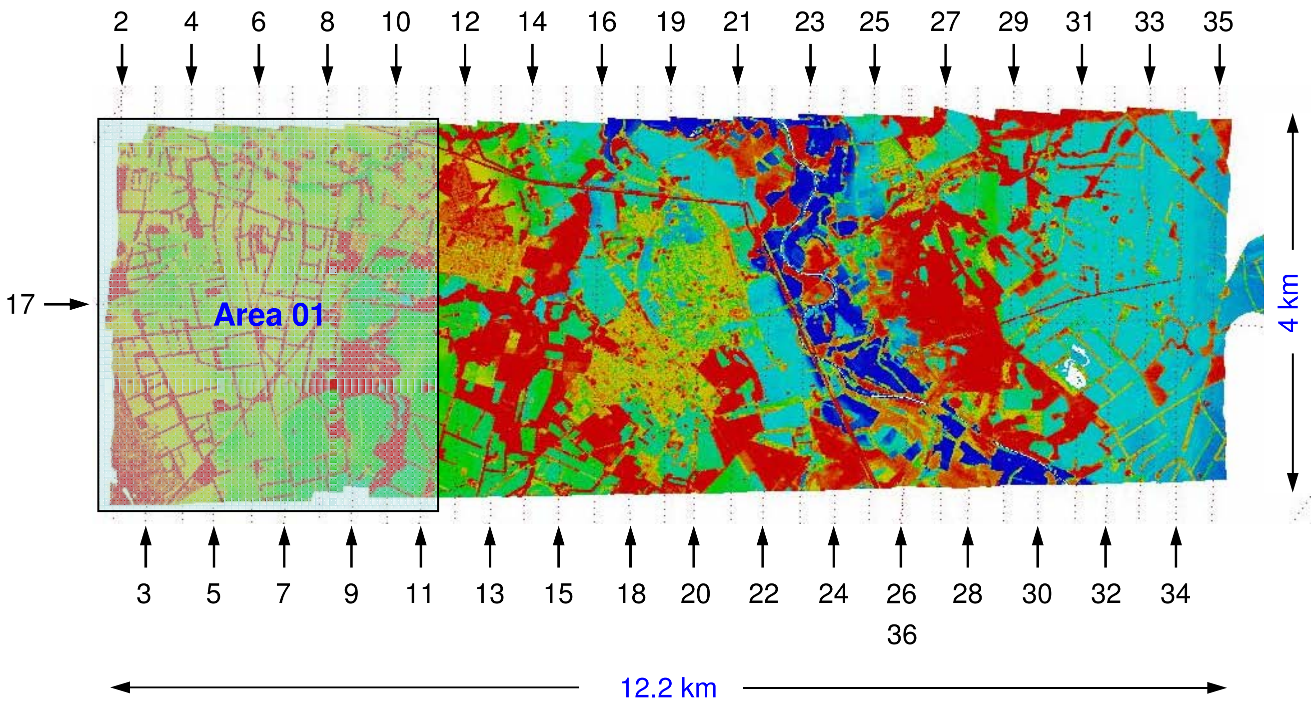
Empirical Test 'Rheine' – Sub-Blocks



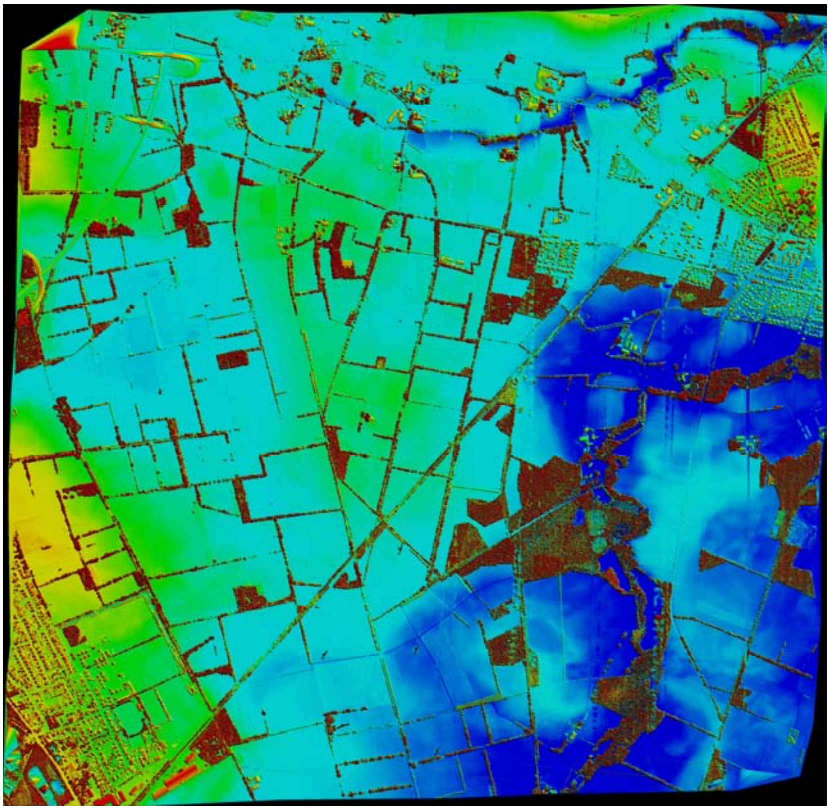
Sub-block **Rheine**

Sub-block **Mesum**

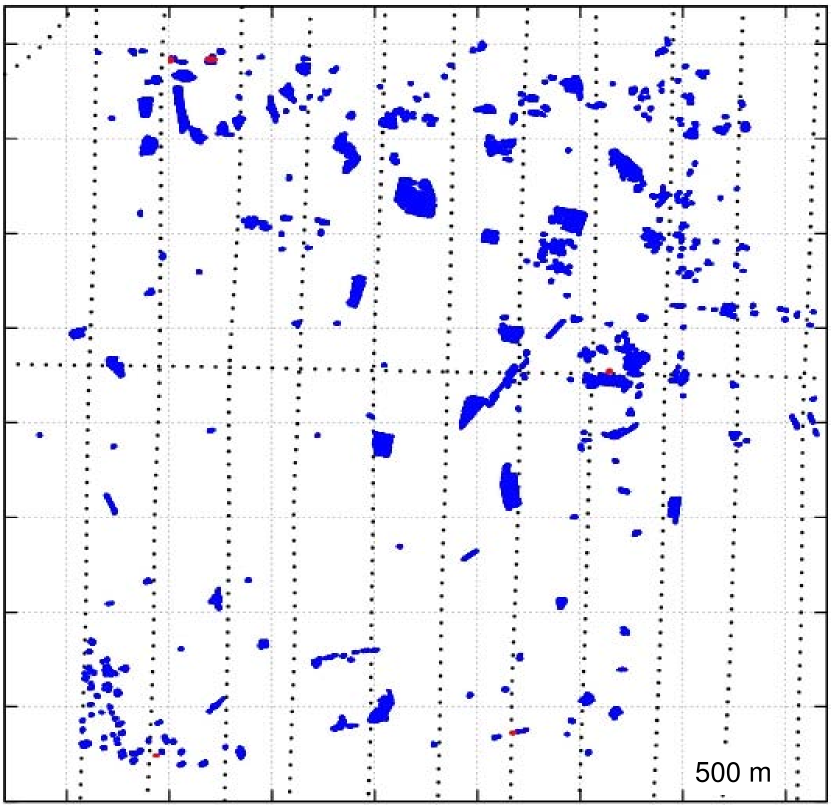
Empirical Test 'Rheine' – Sub-Block Mesum



Empirical Test ‘Rheine’ – Sub-Block Mesum Area 01



Empirical Test ‘Rheine’ – Sub-Block Mesum Area 01



LP Block Adjustment

- 472 planes
- 152 371 plane points
- ~ 322 points/plane

Sub-Block Mesum Area 01 – Unknown Parameters

Observation corrections

Scan angle corrections $\Delta\Theta$

Scan angle corrections Δs

Mounting parameters

ALS-IMU orientation ΔE_Y

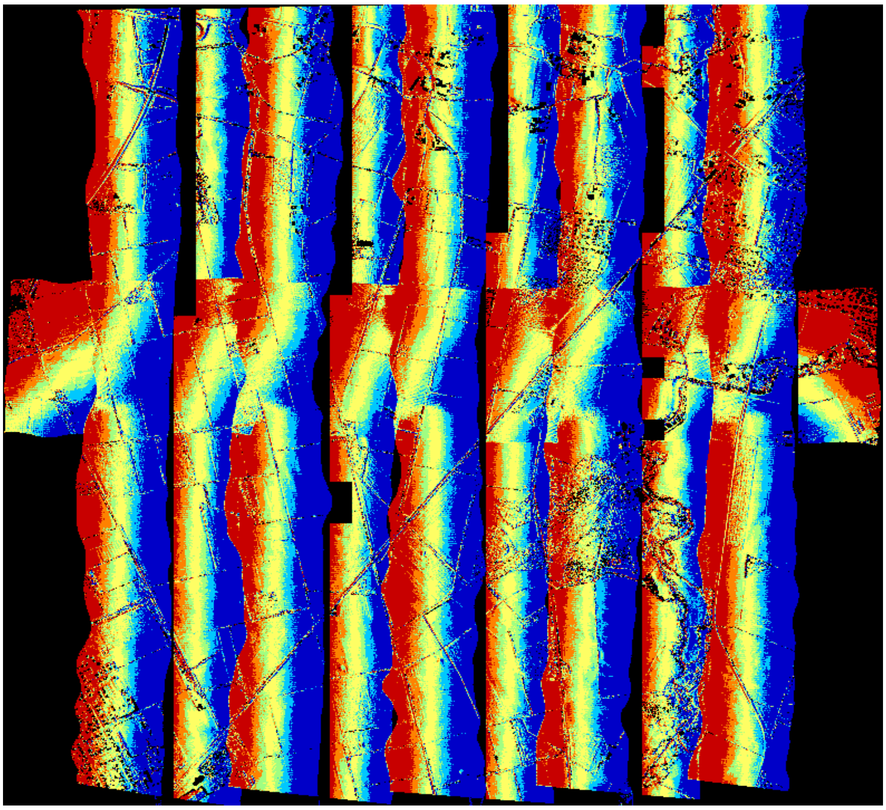
ALS-IMU orientation ΔE_Z

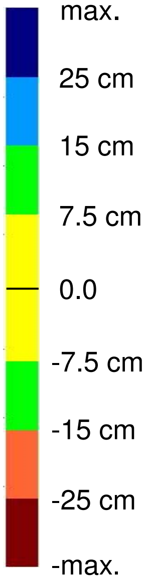
Geo-Referencing corrections per **n-1** flight lines

Position correction Δz_{PC}^{LPF}

⇨ **No control information**

Sub-Block Mesum Area 01 – Unadjusted

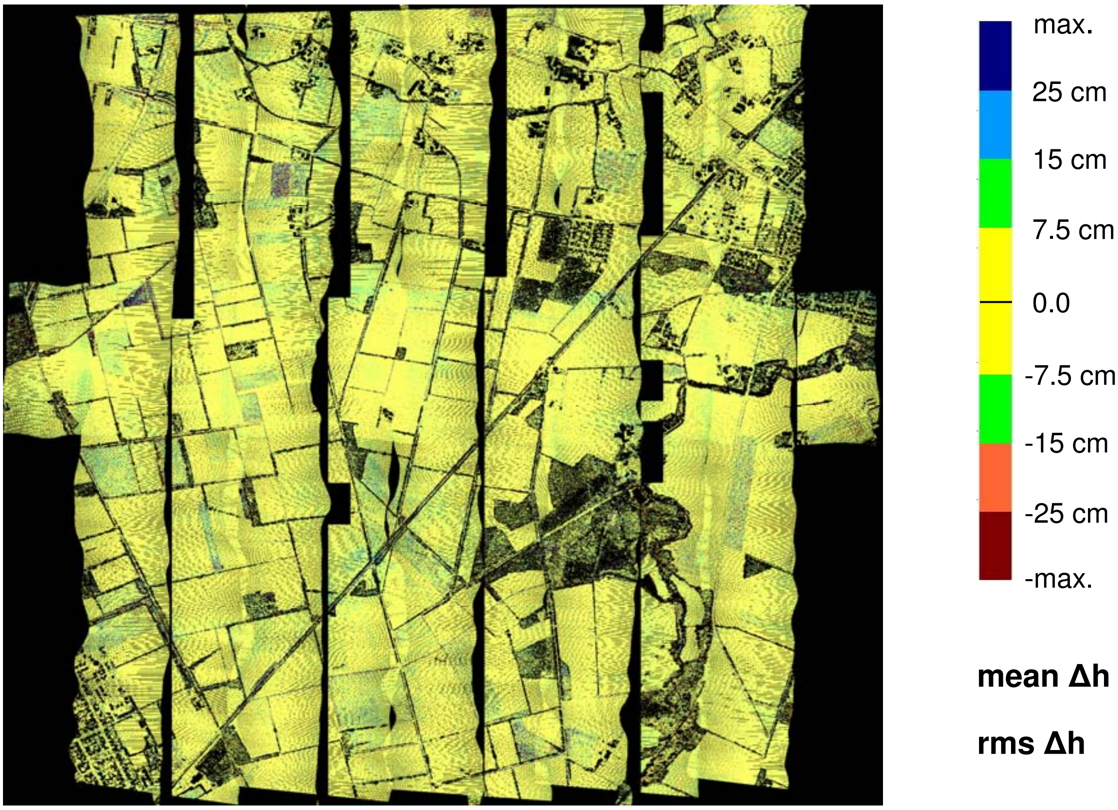




mean Δh = 5.1 cm

rms Δh = 33.0 cm

Sub-Block Mesum Area 01 – Adjusted



mean Δh = 0.2 cm

rms Δh = 6.2 cm

Sub-Block Mesum Area 01 – Parameter Precision

Observation corrections

Scan angle corrections	$\Delta\Theta$	$\hat{\sigma}_{\Delta\Theta}$ =	0.000 07 deg
Scan angle corrections	Δs	$\hat{\sigma}_{\Delta s}$ =	7.7 ppm

Mounting parameters

ALS-IMU orientation	ΔE_Y	$\hat{\sigma}_{\Delta E_Y}$ =	0.000 13 deg
ALS-IMU orientation	ΔE_Z	$\hat{\sigma}_{\Delta E_Z}$ =	0.000 70 deg

Geo-Referencing corrections per **n-1** flight lines

Position correction	Δz_{PC}^{LPF}	$\hat{\sigma}_{\Delta Z}$ =	1.6 – 2.5 mm

Accuracy Verification

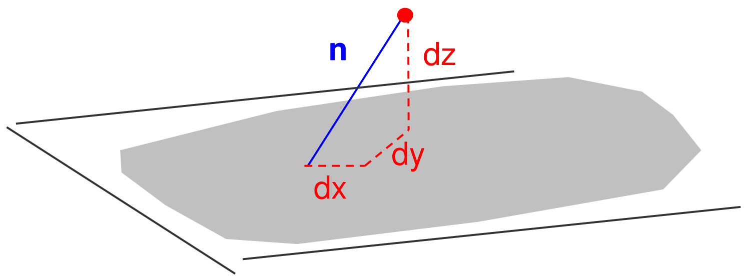
The laser point accuracy can be verified at each of the tie-planes by

- computing the normal vectors **n** for the individual laser points
- deriving statistics for the normal vectors' components and lengths:

mininum dx, dy, dz, ds

maximum dx, dy, dz, ds

rms dx, dy, dz, ds



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